# NOVEMBER 2016 PROFESSIONAL EXAMINATIONS QUANTITATIVE TOOLS IN BUSINESS (PAPER 1.4) CHIEF EXAMINER'S REPORT, QUESTIONS & MARKING SCHEME

## EXAMINER'S GENERAL COMMENTS

The Quantitative Tools in Business, Paper 1.4, written in November was generally well written and the questions adequately cover the Level One Syllabus and the ICAG Manual.

The standard of the Paper was a little weaker, compared to the May 2016 Paper, but it was similar in format and style to the previously administered Papers. The marking scheme was well-drawn; every sub-question had marks duly allocated and the marks were adequate for each question (i.e. it followed the weighting in the revised syllabus). Finally, it was observed that the questions were evenly spread over the topics in the syllabus. The only limitations of the paper are the few <u>misprints</u> found in the sub-questions. However, the marking scheme was straight forward and candidates were rewarded for any meaningful effort.

#### GENERAL PERFORMANCE

The general performance of candidates can be described as below average. Majority of the candidates who wrote the paper at centres outside Accra performed extremely poorly with a few scoring above 40%. The few average performers recorded are found in Accra, Kumasi, Wa and Cape Coast. Indeed the best candidate scored 77% and he wrote the paper at the Accra centre. About a dozen candidates scored between 0% and 5%. There was no traceable copying by candidates except that some candidates decided to write only question numbers into their booklets for examiners to mark. Majority of the candidates also wasted their limited time trying to solve all seven instead of the recommended five <u>questions</u> and ended up scoring very low marks, say, below 10 marks in all seven questions. There were a lot of bad numbering of questions and examiners went through some difficulties trying to separate answered questions for marking and scoring. Per the scripts submitted for marking, one will conclude that candidates' preparation for the paper was very poor and this has reflected in the general performance. In fact, few candidates decided to present blank pages in their booklets for marking in this diet.

#### NOTABLE STRENGTHS & WEAKNESSES

Candidates' notable strengths in the performance were on the regression question (QUESTION THREE), Statistics (QUESTION FIVE). Majority of <u>the</u> candidates could easily do simple calculation of the summary statistics (mean, median, mode, standard deviation, coefficient of variation) and use the calculator very well in the QUESTION, THREE & FIVE. These strengths were demonstrated mostly by candidates who took their paper in Accra, Kumasi, Cape Coast and Wa. This might be due to the availability of teaching and learning materials in these centers, and qualified instructors for teaching the quantitative tools in business

Candidates' main weaknesses were; lack of basic knowledge of algebra and inadequate preparation for the exams. This is reflected in their inability to explain the words ANNUITIES, SINKING FUND and AMORTIZATION. This weakness is widespread and included candidates from even the four averagely high performing centers of Accra, Kumasi, Cape Coast and Wa.

## **QUESTION ONE**

- a) Distinguish between the following terms as used in probability:
  - i) Independents Events and Dependent Events. (2 marks)
  - ii) Mutually exclusive Events and Exhaustive Events. (2 marks)
  - iii) Marginal Probability and Joint Probability. (2 marks)
- b) Mr Agbagba, an ICAG qualified member and his wife an ICAEW qualified member attended an interview for two vacancies for the post of College Finance Officer at a Private University. The probability of the interview panel selecting the man is 1/7 and that of the wife is 1/5.

## **Required:**

Assuming the event selecting a man and selecting a woman are independent, determine the probability that;

i)	both of them will be selected,	(3 marks)
ii)	only one of them will be selected,	(3 marks)
iii)	none of them will be selected.	(3 marks)

c) If selected by the panel the probability that an ICAG qualified member will remain with the Private University is 0.6, and the probability that a Chartered Accountant earns more than GH¢5,000.00 per month in the university is 0.5. If the probability that Mr Agbagba will remain with the university or he will earn more than GH¢5,000.00 per month is 0.7;

## **Required:**

Calculate the probability that he will earn more than GH¢5,000.00 per month given that he is a Chartered Accountant who will stay with the university.

(5 marks)

(Total: 20 marks)

## QUESTION TWO

JinJin Company Limited makes two types of leather belts Type Superior and Type Standard. Type Superior is a highly quality and Type Standard is of lower quality. The respective profits are GHp 40 and GHp 30 per belt. The production of each of Type Superior requires twice as much time as a belt of Type Standard, and if all belts were of Type Standard, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both Type Superior and Type Standard combined). Belt Type Superior requires a fancy buckle and only 400 of these are available per day. There are only 700 buckles a day available for belt Type Standard. If the aim is to determine how many units of the two types of belts the company should manufacture in order to have the maximum overall profit:

## **Required:**

a)	Formulate this problem as a Linear Programming Model.	(4 marks)
b)	Set up the initial Simplex Tableau.	(4 marks)
C)	Solve your Tableau in (b) above.	(8 marks)
d)	Interpret your final Simplex Tableau	(4 marks)

(Total: 20 marks)

#### **QUESTION THREE**

a) The Branch Manager of a building material production plant feels that the demand for Iron rods shipments may be related to the number of construction permits issued in the country during the previous quarter. The Manager has collected the data shown in the table below:

Construction Permits	Iron Rods
15	6
9	4
40	16
20	6
25	13
25	9
15	10
35	16

#### **Required:**

- i) Use the normal equations of the least square regression method to derive a regression forecasting equation for the data (9 marks)
- ii) Interpret your regression coefficient in (i) above. (1 marks)
- iii) Using the regression line in (a) above, determine a point estimate for Iron Rods
- iv) When the number of construction permits is 30. (2 marks)
- v) Is your estimate in (iii) above reliable? Give reason(s) for your answer.

(2 marks)

vi) Calculate the coefficient of determination and interpret it. (6 marks)

(Total: 20 marks)

## QUESTION FOUR

- a) One of the most important application of **annuities** is the repayment of interest- bearing debts. These debts can be paid by making periodic deposits into a **sinking fund** which is used as a future date to pay the principal of the debt or by making periodic payments that cover the outstanding interest and the principle. This second method is called **amortization**.
  - (i) Explain the term **annuities** as used in the statement above. (2 marks)
  - (ii) What is a sinking fund?
  - (iii) When is a loan with a fixed rate of interest said to be amortized?
    - (1 mark)

(2 marks

b) Maame TorTor has just purchased a GH¢70,000.00 house and made a down Payment of GH¢15,000.00.

#### **Required:**

- i) Determine how much money is needed to amortize (.i.e. pay monthly) the balance at 9% interest rate compounded annually for 25 years. (5 marks)
  ii) Determine the total interest for the 25 years. (2 marks)
- iii) Determine after 20 years the equity she has in the house. (3 marks)
- c) Maame TorTor borrows GH¢ 3000.00 and agrees to pay interest quarterly at an annual rate of 8%. At the same time, she set up a sinking fund in order to repay the loan at the end of 5 years. If sinking fund earns interest at the rate of 6% compounded semi-annually,

#### **Required:**

Determine the size of each semi-annual sinking fund deposit. (5 marks)

(Total: 20 marks)

#### **QUESTION FIVE**

Suppose that Mr. Kuu, a retired chartered accountant, is facing a decision about where to invest that small fortune that remains after he has deducted the anticipated expenses for the next year from the earnings from his consultancy job. An investment analyst has suggested to him two types of investment, and to help make the decision he obtained some rates of return from each type. He would like to know what he can expect by way of the return on his investment, as well as other types of information, such as whether the rates are spread out over a wide range (making the investment risky) or are grouped tightly together (indicating a relatively low risk). The returns for the two types of investments are listed here.

Returns on	n Investment	Returns or	Investment B	
30.00	6.93		30.33	-18.75
-2.13	-13.24		-18.37	40.00
4.30	-18.95		-5.61	39.19
25.00	9.43		29.00	-18.23
12.89	1.21		-18.01	4.16
-18.24	31.76		0.46	10.03
1.20	11.07		2.07	10.51
-2.59	8.47		29.44	39.04
33.00	36.08		11.00	24.76
14.26	-18.95		-18.93	15.28

If he decides to group the returns according to classes 19-10, 9-0,1-10, 11-20,21-30,31-40:

#### **Required:**

a)	Draw histograms for each set of returns.	(5 marks)
b)	Compute the following measures of central tendency for the set of i) Mean ii) Median iii) Mode	returns: (3 marks) (3 marks) (3 marks)
c)	Compute the following measures of spread for the set of returns: i) Standard deviation	(3 marks)

ii) Coefficient of variation (2 marks)

d) Using (a), (b) and (c) above, which investment should Mr.Kuu choose and why? (1 marks)

(Total: 20 marks)

## **QUESTION SIX**

a) Sketch Graph(s) to show the following critical points on a cost or revenue function:

i) Local Maximum Point	(1 mark)
ii) Absolute Maximum Point	(1 mark)
iii) Local Minimum Point	(1 mark)
iv) Absolute Minimum Point	(1 mark)
v) Point of inflexion	(2 marks)

b) Tema Steel Plant is capable of producing  $q_1$  tons per day of a low grade steel and  $q_2$  tons per day of a high grade steel, where  $q_2 = \frac{40-5q_1}{10-q_1}$ . If the fixed market price of low grade steel is GH¢ 6.90 and the fixed market price of high grade steel is GH¢ 13.80.

## **Required:**

i) Determine the number tons of low grade steel and high grade steel to be produced to maximize total revenue (10 marks)

ii) Determine the maximum total revenue. (4 marks)

(Total: 20 marks)

#### **QUESTION SEVEN**

The personnel department of Dropper Ltd, a large cocoa processing company in DropperLand, is concerned about absenteeism among its shop floor workforce. There is a general feeling that the underlying trend has been rising, but nobody has yet analysed the figures. The total number of shops employees has remained virtually unchanged over the few years.

The mean number of absentees per day for each quarter of the years 1999 to 2001 and quarter 1 in 2002 is given in the table below:

Mean Number of Absentees								
	Q1	Q2	Q3	Q4				
1999	25.10	14.40	9.50	23.70				
2000	27.90	16.90	12.40	26.10				
2001	31.40	19.70	15.90	29.90				
2002	34.50							

## **Required:**

- a) Plot the data on a graph, leaving space for the remaining 2002 figures.
- b) Using the method of 2 quarterly centered moving,
  - i) determine the trend in the series and superimpose this on your graph in (a) above. (4 marks)
  - ii) determine the equation of the trend line above by considering only the first and last centered moving average value on your graph in (i) above.

(3 marks)

(3 marks)

- c) Using an appropriate decomposition model to determine the seasonal variations in the data. Give reasons for your choice of model. (5 marks)
- d) Use your analysis above to produce roughly forecasts of the mean number of absentees there will be in the remaining quarters of 2002. Comment on your forecast.
   (5 marks)

(Total: 20 marks)

#### **QUESTION ONE**

a)

i) Two events said to be **independent** if the occurrence ( or non-occurrence ) of one will not in any way affect the occurrence ( or non-occurrence) of the other.
 Two events are said to **dependent** if the occurrence or (non-occurrence) of

Two events are said to **dependent** if the occurrence or (non-occurrence) of one will affect the occurrence (or non-occurrence) of their other.

- ii) Two events are **mutually exclusive** if both events cannot occur together in a random experiment. They are said to be **exhaustive events** if they are complementary events.
- iii) The **joint probability** of two events is the probability that both events will occur together in an experiment. The probability of having one of the two events is term the **marginal probability** of the event.
- b)
  - i) Let *A* be the event a man is selected and *B* be the event a woman is selected. Then,

$$P(A) = \frac{1}{7}, P(B) = \frac{1}{5}$$

$$P(A \cap B) = P(A)P(B) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$$
(3 marks)

ii) 
$$P(A \cap B^{c}) + P(A^{c} \cap B) = P(A)P(B^{c}) + P(A^{c})P(B) = \frac{1}{7} \times \left(1 - \frac{1}{5}\right) + \left(1 - \frac{1}{7}\right) \times \frac{1}{5}$$
$$= \frac{4}{35} + \frac{6}{35} = \frac{2}{7}$$

(3 marks)

iii) 
$$P(A^c \cap B^c) = \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{5}\right) = \frac{24}{35}$$

c) Let ICAG be the event chartered account will stay with the university and let GHC5000 be the event a chartered accountant earns more than GHC 5000 Ghana Cedis.

 $P(GHC5000 | ICAG) = \frac{P(GHC5000 \cap ICAG)}{P(ICAG)}$ 0.7 = 0.5 + 0.6 - P(GHC5000 \cap ICAG) P(GHC5000 \cap ICAG) = 1.1 - 0.7 = 0.4

 $P(GHC5000 | ICAG) = \frac{0.4}{0.6} = \frac{2}{3}$  (5 marks) (Total: 20 marks)

#### EXAMINER'S COMMENTS

It was <u>a</u> less popular question among candidates but many candidates who answered it scored extremely low marks (i.e. 0/20,1/20,2/20). Only a handful of candidates could distinguish between the terms INDEPENDENT EVENTS and DEPENDENTS EVENTS, MUTUALLY EXCLUSIVE EVENTS and EXHAUSTIVE EVENTS, MARGINAL PROBABILITY AND JOINT PROBABILITY. In Sub-section (c)<sub>*t*</sub> the question <u>on</u> conditional probability was a challenge to many students. Please refer to the marking scheme for the best approach to answer it. The highest mark in this question is 15

#### **QUESTION TWO**

a) Let  $s_1$  and  $s_2$  be number of belts of types Superior and Standard respectively. Objective function

Max  $Z = 0.4s_1 + 0.3s_2$ Subject to constraints  $2s_1 + s_2 \le 1000$   $s_1 + s_2 \le 800$   $s_1 \le 400$   $s_2 \le 700$  $s_1 \ge 0, s_2 \ge 0$  (4 marks)

b)

Initial Simplex Tableau

	minur omplex rubleuu								
Solution	Decisi	ion	<u> </u>	Slack Variables					
Variable	Varia	ble							
	$S_1$	<i>s</i> <sub>2</sub>	$x_1$	$x_2$	<i>x</i> <sub>3</sub>	$x_4$			
<i>x</i> <sub>1</sub>	2	1	1	0	0	0	1000		
<i>x</i> <sub>2</sub>	1	1	0	1	0	0	800		
<i>x</i> <sub>3</sub>	1	0	0	0	1	0	400		
$X_4$	0	1	0	0	0	1	700		
Z.	0.4	0.3	0	0	0	0	0		

(4 marks)

1000/2=500 800/1=800 400/1=400

Solution	Decis	ion	Slack Variables				Solution	
Variable	Varia	ble					quantity	
	S <sub>1</sub>	<i>s</i> <sub>2</sub>	$x_1$	$x_2$	<i>x</i> <sub>3</sub>	$X_4$		
<i>x</i> <sub>1</sub>	0	1	1	0	-2	0	200	
<i>x</i> <sub>2</sub>	0	1	0	1	-1	0	400	
<i>S</i> <sub>1</sub>	1	0	0	0	1	0	400	
<i>x</i> <sub>4</sub>	0	1	0	0	0	1	700	
Z.	0	0.3	0	0	-	0	-160	
					0.4			

#### First Iteration

# Second Iteration

Solution Variable	Decis Varia	ion ble	Slack Variables				Solution
Variable	s <sub>1</sub>	s <sub>2</sub>	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>x</i> <sub>3</sub>	$X_4$	quantity
<i>s</i> <sub>2</sub>	0	1	1	0	-2	0	200
<i>x</i> <sub>2</sub>	0	0	-1	1	1	0	200
<i>S</i> <sub>1</sub>	1	0	0	0	1	0	400
<i>x</i> <sub>4</sub>	0	0	-1	0	2	1	500
Z.	0	0	-0.3	0	0.2	0	-220

Third	Iteration

Solution Variable	Decision Slack Variables Variable					Solution quantity	
	<i>s</i> <sub>1</sub>	<i>s</i> <sub>2</sub>	$x_1$	$x_2$	<i>x</i> <sub>3</sub>	$X_4$	
<i>s</i> <sub>2</sub>	0	1	-1	2	0	0	600
<i>x</i> <sub>3</sub>	0	0	-1	1	1	0	200
S <sub>1</sub>	1	0	1	-1	0	0	200
<i>x</i> <sub>4</sub>	0	0	1	-2	0	1	100
z	0	0	-0.1	-0.2	0	0	-260

(8 marks)

c)

d) The company should produce 200 of Type Superior and 600 of Type Standard, and the maximum profit would be 260. 200 and 100 in the solution quantity are excess capacity in leather and buckles constraint respectively. The shadow prices for the resources are 0.1, 0.2, 0, and 0 respectively.

(4 marks)

## (Total: 20 marks)

#### EXAMINER'S COMMENTS

This question was also a less popular choice among candidates, and those who answered it did not do well  $_{\perp}$  with the average mark hovering around the figure 7/20  $_{7}$  with one candidate scoring 13/20. Candidates who attempted this question could not formulate the linear programming model and only a few could manage some iteration.

#### **QUESTION THREE**

a)	
i)	

Permits	Rods	xy	$x^2$	$y^2$
(x)	(y)			-
15	6	90	225	36
9	4	36	81	16
40	16	640	1600	256
20	6	120	400	36
25	13	325	625	169
25	9	225	625	81
15	10	150	225	100
35	16	560	175	256
$\sum x = 144$	$\sum y = 60$	$\sum xy = 2146$	$\sum x^2 = 3,956$	$\sum y^2 = 950$

$$8b_{0} + 144b_{1} = 60$$

$$144b_{0} + 3956b_{1} = 2146$$

$$b_{0} = \frac{\begin{vmatrix} 60 & 144 \\ 2146 & 3956 \end{vmatrix}}{\begin{vmatrix} 8 & 144 \\ 144 & 3956 \end{vmatrix}} = \frac{60 \times 3956 - 2146 \times 144}{8 \times 3956 - 144 \times 144} = \frac{237,360 - 309,024}{31,648 - 20,736} = \frac{-71,664}{10,912} = -6.57$$

$$b_{1} = \frac{\begin{vmatrix} 8 & 60 \\ 144 & 2146 \\ 8 & 144 \\ 144 & 3956 \end{vmatrix}}{\begin{vmatrix} 8 & 144 \\ 144 & 3956 \end{vmatrix}} = \frac{8 \times 2146 - 144 \times 60}{10912} = \frac{17,168 - 8,640}{10912} = \frac{8528}{10912} = 0.78$$
  
$$y = -0.67 + 0.78x$$
 (9 marks)

ii) On the average a permit issue will result in 0.78 Iron Rod been demanded. (1 mark)

iii) 
$$y = -0.67 + 0.78(30) = -0.67 + 23.4 = 22.73$$

iv) The point estimate in (i) above is reliable because 30 is within the range of the data. (2 marks)

v) 
$$R = \frac{(8 \times 2146 - 144 \times 60)^2}{\left[8 \times 3956 - (144)^2\right] \left[8 \times 950 - (60)^2\right]} = \frac{(17168 - 8640)^2}{\left[31648 - 20736\right] \left[7600 - 3600\right]} = \frac{(8528)^2}{10,912 \times 4000} = 0.2\%$$

Roughly 0.2 percent of the variations in demand of Iron rod is explained by changes in building permit.

(6 marks) (Total: 20 marks)

(2 marks)

#### **EXAMINER'S COMMENTS**

<u>Interestingly</u>, Question Three was the most popular question among candidates after Question Five . <u>One</u> candidate scored 20/20. However, few candidates could manage 12 marks and above. Almost all the candidates who took this diet's Paper 1.4 attempted this question.

#### **QUESTION FOUR**

- a)
- i) Annuity is a sequence of fixed annual payments (or receipts) made at uniform (or equal) time interval. (2 marks)
- ii) **Sinking fund** is amount set aside plus interest to retire or pay a n interest bearing debt. (2 marks)
- iii) A loan with a fixed rate of interest is said to be amortized if both principle and interest are paid by a sequence of equal payments made over equal periods of time. (1 mark)

b)

i) The monthly payment P needed to pay off the loan of GHC 55,000 at 9% interest per annum for 5 years is given as

$$P = GHC55,000 \left( \frac{1}{\left[ 1 - (1 + 0.0075)^{-300} \right] / 0.075} \right) = GHC55,000 (0.008392)$$
  
= GHC461.56 (5 marks)

ii) Total Interest on the loan = 
$$461.56(300) - 55,000 = GHC83,468.00$$
 (2 marks)  
iii) After 5 years (60 months), the present value of the loan is  
 $GHC461.56 \times [1 - (1 + 0.0075)^{-60}] / 0.075 = GHC461.56 \times 48.17$   
 $= GHC22,234.90$   
Thus, the equity after 5 years is GHC 55,000-22,234.90=GHC 32,765.10  
(3 marks)

c) The quarterly interest payments due on the debt are GHC 3000(0.2)=GHC 60 The size of each semi-annual deposit P is given by P=

$$3000 \left[ \frac{(1+0.03)^{10} - 1}{0.03} \right] = 3000(0.0872) = GHC261.69$$

(5 marks)

(Total: 20 marks)

#### **EXAMINER'S COMMENTS**

Candidates who answered this question performed badly. In fact, they could not explain the terms <u>ANNUITIES, SINKING FUND AND AMORTIZATION</u> Candidates could not apply the concepts of discounting to solve problems in mathematics of finance. In fact  $_{\perp}$  candidates who <u>want</u> to answer questions on mathematical finance are entreated to take their lessons on series and sequence seriously. Some candidates however managed to get between 14/20 and 15/20 in this question.

# **QUESTION FIVE**

## a)

Group Frequency Table For Returns on Investment A

Class		Class	Class	fx	$fx^2$
	Frequency(f)	Boundary	Midpoint(x)		v
19–10	4	19.5–9.5	-14.5	-58.0	841.0
9-0	2	9.5-0.5	-4.5	- 9.0	40.5
1-10	6	0.5-10.5	5.5	33.0	181.5
11-20	3	10.5-20.5	15.5	46.5	720.8
21-30	2	20.5-30.5	25.5	51.0	1300.5
31-40	3	30.5-40.5	35.5	106.5	3,780.8
	$\sum f = 20$			$\sum fx =$	$\sum fx^2 =$
				170	6865.1

# Group Frequency Table For Returns on Investment B

Class		Class	Class	fx	$fx^2$
	Frequency(f)	Boundary	Midpoint(x)		·
19–10	5	19.5–9.5	-14.5	-72.5	1051.3
9-0	1	9.5-0.5	-4.5	-4.5	20.3
1-10	3	0.5-10.5	5.5	16.5	90.8
11-20	4	10.5 - 20.5	15.5	62.0	961.0
21-30	3	20.5-30.5	25.5	76.5	1950.8
31-40	4	30.5-40.5	35.5	142	5041.00
	$\sum f = 20$			$\sum fx =$	$\sum fx^2 =$
				220	9115.2





i) Mean(Return A) = 
$$\frac{170}{20}$$
 = 8.5  
Mean (Return B) =  $\frac{220}{20}$  = 11 (3 marks)

ii) Mode (Return A)=
$$0.5 + \left(\frac{2}{2+3}\right) \times 10 = 5.5$$
  
Mode (Return B) =  $-19.5 + \left(\frac{0}{0+1}\right) \times 10 = -19.5$  (3 marks)

iii) Median(Return A)=
$$0.5 + \left(\frac{\left[20/2\right]-6}{6}\right) \times 10 = 7.17$$
  
Median (Return B) = $10.5 + \left[\frac{(20/2)-8}{4}\right] \times 10 = 15.5$  (3 marks)

c)

i) Standard deviation (Return A) = 
$$\sqrt{\frac{\left[6865.1 - (170)^2 / 20\right]}{20 - 1}} = \sqrt{271.01} = 16.46$$

Standard deviation (Return B) = 
$$\sqrt{\frac{9115.2 - (220)^2 / 20}{20 - 1}} = \sqrt{334.76} = 18.30$$
  
(3 marks)

ii) Coefficient of Variation (Return A) = 
$$\frac{16.46}{8.50} = 193.65\%$$
  
Coefficient of Variation (Return B) =  $\frac{18.30}{11} = 166.36\%$ 

(2 marks)

d) Mr Kuu should choose investment B because it is less risky and the returns it is higher. (1 mark) (Total: 20 marks)

# EXAMINER'S COMMENTS

This question was the next most popular after <u>Question Three</u>. Candidates were wellprepared for statistics questions. However, many candidates could not work with negative numbers. In particular, forming class boundaries was a problem for candidates. Candidates who could form the class boundaries could calculate the summary measures and draw the histogram. The only limitation identified <u>is</u> found in subsection (c) where candidates could not give a good comment. However, there were very good answers from few candidates from the four centers with some scoring 15/20, 16/20 and 17/20 respectively.



i) Total Revenue is given by  $TR(q_1) = 6.90q_1 + 13.80q_2 = 6.90q_1 + 13.80\left(\frac{40 - 5q_1}{10 - q_1}\right)$ 

At the point of maximum revenue we have that

$$\frac{dTR}{dq_1} = 6.90 + 13.80 \left[ \frac{(10 - q_1)(-5) - (40 - 5q_1)(-1)}{(10 - q_1)^2} \right] = 0$$
  
Solving for  $q_1$  we have  
 $-50 + 5q_1 + 40 - 5q_1 = -0.5(10 - q_1)^2$   
 $(10 - q_1)^2 = 20$   
 $10 - q_1 = \pm\sqrt{20}$   
 $-q_1 = -10 \pm \sqrt{20}$   
 $q_1 = 10 + \sqrt{20} = 10 + 4.4721 = 14.4721$  or  
 $q_1 = 10 - \sqrt{20} = 10 - 4.4721 = 5.5279$   
Now  $TR(5.5279) = 6.90(5.5279) + 13.80 \left( \frac{40 - 5(5.5279)}{10 - (5.5279)} \right)$ 

$$TR(5.5279) = 38.14251 + 13.80 \left(\frac{40 - 27.6395}{10 - 5.5279}\right) \approx 38.142 + 38.142 = 76.284$$

Also

$$TR(14.4721) = 6.90(14.4721) + 13.80\left(\frac{40 - 5(14.4721)}{10 - 14.4721}\right) \approx 99.857 + 99.857 = 199.71$$
  
AS  $TR(14.4721) > TR(5.5279)$  we have that  $q_1 = 10 + \sqrt{20} = 10 + 4.4721 = 14.4721$ 

and  $q_2 = \frac{40 - 5(14.4721)}{10 - 14.4721} = 7.236$  give the maximum revenue.

#### (10 marks)

ii) Maximum total revenue is  

$$TR(14.4721) = 6.90(14.4721) + 13.80 \left(\frac{40 - 5(14.4721)}{10 - 14.4721}\right) \approx 99.857 + 99.857 = 199.71$$
(4 marks)

(Total: 20 marks)

#### **EXAMINER'S COMMENTS**

Question Six <u>was</u> one of the least popular questions among candidates but most candidates who attempted this question <u>performed poorly</u>. Some candidates scored as low as 0 marks in this question. The effect of candidates' very limited knowledge in Algebra was exhibited in the answers they provided. Only a handful could manage with the computation of profit from the cost and revenue functions. I recommend candidates master the theory part of the topics because their understanding of the applied part of questions are dependent on the theory

b)

# **QUESTION SEVEN**



(3 marks)

Quarters	Number	2 Quarterly	2 Quarterly	Trendline	Seasonal
	of	M. A.	Centered		Variation
	Absentees		M.A.		
1	25.10			14.55	10.55
		19.50			
2	14.40		15.73	15.73	-1.33
		11.95			
3	9.50		14.28	16.91	-7.41
		16.60			
4	23.70		21.20	18.09	5.61
		25.80			
5	27.90		24.10	19.27	8.63
		22.40			
6	16.90		18.33	20.45	-3.45
		14.65			
7	12.40		16.95	21.63	-9.23
		19.25			
8	26.10		24.00	22.81	3.29
		28.75			
9	31.40		27.15	23.99	7.41
		25.55			
10	19.70		21.68	25.17	-5.47
		17.80			
11	15.90		20.35	26.35	-10.45
		22.90			
12	29.90		27.55	27.53	2.37
		32.2			
13	34.50			28.71	5.79

## (4 marks)

ii) Gradient of trendline is given by  $b = \frac{27.55 - 15.73}{12 - 2} = 1.182$ . Therefore the trendline base on the 2 quarterly centered moving average is given by y = 15.73 + 1.18x, where x = -1, 0, 2, 3, ..., 10, 11, 12, 13, 14

(3 marks)

c)					
	SEASONAL VARIATION				
		1			
	Q1	Q2	Q3	Q4	TOTAL
YEAR					
1999	10.55	-1.33	-7.41	5.61	
2000	8.63	-3.43	-9.23	3.29	
2001	7.41	-5.47	-10.45	2.37	
2002	5.79				
TOTAL	32.38	-10.23	-27.1	11.27	
AVERAGE	16.19	-3.41	-9.03	3.76	-7.51
ADJUSTMENT	-1.8775	-1.8775	-1.8775	-1.8775	
FACTOR					
ADJUSTED	14.31	-5.29	-10.90	1.88	0
AVERAGE					

REASON: I use the additive model because from the graph in (a) above seasonal variation are not affected by the increasing trend factors.

## (5 marks)

d)			
2002		AVERAGE	SEASONALLY
	TRENDLINE	SEASONAL	ADJUSTED
		VARIATION	FORECAST
Q2	29.89	-5.29	24.6
Q3	31.07	-10.90	20.17
Q4	32.25	1.88	34.13

Comment: These forecasts are not reliable because they are extrapolations.

(5 marks)

## (Total: 20 marks)

## EXAMINER'S COMMENTS

Answers provided by candidates for Question Seven (a) were not appropriate. Candidates could not calculate and plot centered moving average. Time series decomposition was a problem and performing an extrapolation was a major problem for candidates. Some candidates scored 0/20 in this question.