MAY 2019 PROFESSIONAL EXAMINATIONS QUANTITATIVE TOOLS IN BUSINESS (PAPER 1.4) CHIEF EXAMINER'S REPORT, QUESTIONS AND MARKING SCHEME

EXAMINER'S GENERAL COMMENTS

The Quantitative Tools in Business, Paper 1.4, taken in May 2019 was well written. The standard of the Paper was a little higher than that of the November 2018, and the format and style were similar to previous examination papers. There were limited ambiguities in the paper. The marking scheme was well- drawn up; every sub-question had marks duly allocated, and the marks were adequate for each question.

GENERAL PERFORMANCE

The general performance of candidates can be described as below average. Majority of candidates performed poorly, with many scoring below 45%. A few candidates scored below 10%. There were no traceable copying by candidates, except that some candidates did not number their answers very well, which made some examiners have serious challenges trying to separate answered questions for marking and scoring. Many candidates attempted the recommended five questions, but scored very low marks on as many as four questions. Per the scripts submitted for marking in May 2019 Examination diet, one will infer that candidates' preparation for the paper was highly inadequate as compared to previous diets, and this is reflected in the general performance of candidates. No student could score 20/20 in any of the questions answered. The best candidate scored about 73% and the worse candidate could manage only about 4%.

NOTABLE STRENGTHS & WEAKNESSES

Candidates' notable strengths in the performance were as usual on Forecasting (QUESTIONS ONE & FIVE), and probability (QUESTION SIX). Many candidates could easily compute moving averages and decompose the time series. Further, computation of probabilities in Question six was well carried out by many candidates who attempted them. The less patronized questions were QUESTIONS THREE & SEVEN.

Candidates' main weaknesses were: lack of basic knowledge of mathematical concepts and inadequate preparation by some candidates for the exams. This is reflected in their inability to explain simple terms such as Internal Rate of Return, Net Present Value, Collectively exhaustive events and complement of an event.

QUESTION ONE

The number of clients who consulted Tsoo Consult within a period of three years were recorded as follows:

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1	75	70	75	80
2	95	85	80	65
3	100	105	115	90

Required:

- a) Assuming a 4 quarterly cycle, calculate the centred trend values for the data by moving average method. (4 marks)
- b) Using (a) above and the multiplicative model, calculate the average seasonal variations. (5 marks)
- c) Using (b) above, calculate the adjusted average seasonal variations for the data. (5 marks)
- d) Using the trend and the adjusted average seasonal variation, forecast the number of clients for Year 4 based on the multiplicative model. (6 marks)

(Total: 20 marks)

QUESTION TWO

- a) Distinguish between *Internal Rate of Return* and *Net Present Value*. (4 marks)
- b) BonBone Company Ltd wants to make a decision on which of the two machines to purchase. Each will involve a GH¢10,000 investment. The expected net incremental cash flows are given by the table below:

Year	Machine I (GH¢)	Machine II (GH¢)
1	5,000.00	2,000.00
2	4,000.00	3,000.00
3	2,000.00	5,000.00
4	2,000.00	4,000.00

Required:

- i) If the company's cost of capital is 10%, calculate the Net Present Value (NPV) of Machine I and Machine II and determine which machine should be purchased for higher returns
 (8 marks)
- ii) If the initial investment for Machine I is changed to GH¢4,000 and Machine II is changed to GH¢2,000, calculate the Internal Rate of Return (IRR) for Machine I and Machine II.
 (6 marks)
- iii) If the IRR's in (ii) above are to be used as the basis of selection, determine which machine should be purchased for higher returns. (2 marks)

QUESTION THREE

The Business Manager of Omaya Art Gallery has rented a hall to display the artworks of the artists of the gallery. She is considering organizing an exhibition of a number of rare painting masterpieces. In the past, only 70% of the paintings were sold in the first week. Moreover, if no painting is sold in the first five (5) days, the exhibition could be extended for another two (2) days but only 20% of the paintings would be sold.

The cost of the exhibition is $GH\phi500$ per day. The manager estimated that in case she does not make any sales she will have to pay $GH\phi15,000$ to cover the costs of renting the exhibition hall for the same period.

Required:

a) Draw a decision tree representing the Business Manager's decision making process.

b)	Calculate the expected monetary cost of each decision node.	(8 marks) (6 marks)
c)	Determine the Business Manager's optimal strategy.	(6 marks)
		(Total: 20 marks)

QUESTION FOUR

a) The continuous random variable X is normally distributed with mean μ and variance σ^2 .

Required:

Sketch the distribution of X and indicate on your sketch the *mean, median and mode*.

(4 marks)

b) CarJoy Manufacturing Company produces electronic components which have life spans that are normally distributed with mean, μ hours and standard deviation, σ hours. Only 1% of the components have a life span less than 3,500 hours and 2.5% have a life span greater than 5,500 hours.

Required:

- i) By standardizing the values of X to the standard normal values, obtain TWO (2) linear equations in μ and σ . (4 marks)
- ii) Using an appropriate method of solving simultaneous equations, determine the value of μ and value of σ in (i) above. (4 marks)
- iii) Determine the proportion of electronic components with life spans between 4,120.50 hours and 5,052.45 hours. (4 marks)
- iv) If the company gives warranty of 4,000 hours on the components, find the percentage of components the company is expected to replace under the warranty. (4 marks)

QUESTION FIVE

a) The objective of smoothing methods is to smoothen out the random variations due to irregular components of the time series and thereby provides an overall impression of the pattern of movement in the data over time.

Required:

Explain the following smoothing methods:

- i) Moving averages
- ii) Exponential Smoothing

- (2 marks) (2 marks)
- b) The number of enquiries being made to a mail order business during Monday to Friday working week is given as:

Week	Monday	Tuesday	Wednesday	Thursday	Friday
1	34	36	24	25	41
2	33	34	24	23	43
3	35	37	25	25	47

Required:

- i) Plot the data on a graph.
- ii) Compute a 5 period moving average values for the data.
- iii) Compute a 'weighted' moving average values for the data if the smoothing Constant is $\alpha = 0.5$.
- iv) Superimpose the graphs of (ii) and (iii) on your graph in (i) above.
- v) Comment on the suitability of the two smoothing methods above.

(16 marks)

QUESTION SIX

- a) Define the following terms in probability theory:
- i) Collectively exhaustive events.
- ii) The complement of an event.
- b) A survey of The Institute of Chartered Accountants (Ghana) students asked the question 'What is your favourite sport?' The results are summarised below:

Level	Football	Boxing	Hockey	Total
1	68	41	46	155
2	84	56	70	210
3	59	74	47	180
Total	211	171	163	545

Required:

- i) What is the probability of selecting a student whose favourite sport is boxing? (2 marks)
- ii) What is the probability of selecting a Level 1 student?
- iii) If the student selected is a Level 2 student, what is the probability that the student prefers hockey? (3 marks)
- iv) If the student selected is a Level 2 student, what is the probability that the student prefers football or hockey? (3 marks)
- v) If the student selected prefers football, what is the probability that the student is a Level 1 student? (3 marks)
- vi) If the student selected is a Level 3 student, what is the probability that the student prefers football, boxing or hockey? (3 marks)

(Total: 20 marks)

(2 marks) (2 marks)

(2 marks)

QUESTION SEVEN

a) Joycarpap Ltd manufactures and sells three models of affordable toys: Car, Joy and Pap. Each model requires specific amount of fabrication, material worth and assembly hours, as shown in the table below:

	Models		
Requirements	Car	Joy	Pap
Fabrication (Hrs)	5 hours	6 hours	4 hours
Materials Worth	GH¢10	GH¢8	GH¢5
Assembly (Hrs)	4 hours	2 hours	5 hours

There are 210 fabrication hours available and 170 hours of assembly. Materials in stock are worth GH¢200. Market research conducted by the company revealed that demand for the toys was such that in whatever combination of the three models produced, all of the output can be sold within a week.

Each Car contributes GH¢15 to profit, each Joy contributes GH¢20 to profit and each Pap contributes GH¢14 to profit. Using, $X_{1,} X_{2,} X_{3}$ as decision variables, $S_{1,} S_{2,} S_{3}$ as slack variables and P as total profit,

Required:

i)	Formulate a linear programming problem.	(4 marks)
ii)	Set up the initial Simplex Tableau.	(4 marks)
iii)	Determine the total profit in the first iteration	(5 marks)

b)

i) Any entity that issues a bond to raise capital would need to pay off the bond when it matures. Paying the debt early via a sinking fund saves a company interest expense and prevents the company from being put in financial difficulties in the future if economic or financial conditions worsen.

Required:

What is Sinking Fund?

ii) The owner of a Business Centre purchased a robust photocopier for serving the UG University Students Community. The photocopier is expected to be replaced after 10 years. He therefore decided to set up a sinking fund and pay an equal annual amount to realise GH¢50,000 being the replacement cost.

Required:

Compute the equal annual amount he should invest if interest rate per annum is 10%.

(4 marks)

(3 marks)

SOLUTION TO QUESTIONS

QUESTION ONE

(a & b) Period, t	Clients, y	4QMT	8QMT	4CMA, T	$S = Y \underline{*100}$ T
1	75				
2	70	300			
3	75	320	620	77.50	90.32
4	80	335	655	81.88	97.70
5	95	340	675	84.38	112.59
6	85	225	665	83.13	102.25
7	80	320	655	81.88	97.70
8	65	350	680	85.00	76.47
9	100	385	735	91.88	108.84
10	105	410	795	99.38	105.66
11	115	410			
12	90				

(9 marks)

c)

	Quarter 1	Quarter 2	Quarter 3	Quarter 4	
			90.32	97.70	
	112.59	102.25	97.70	76.47	
	108.84	105.66	-	-	
Sum	221.43	207.91	188.02	174.17	
Average	110.72	103.96	94.01	87.09	395.78

Adjustment	110.72 <u>X</u> 400	103.9 <u>6 X</u> 400	94.01 X 400	87.09 X 400	
Factor	395.78	395.78	395.78	395.78	
Adjusted	111.90	105.07	95.01	88.02	400
Seasonal					
Index					

(5 marks)

d)

 $b = \underline{99.38 - 77.50} = 3.126$ 10 - 3

T = 77.5 + 3.126t

Period	T= 77.5 + 3.126t	Seasonal Index	Forecast
13	109	119.0%	130
14	112	105.07%	118
15	115	95.01%	109
16	118	88.02%	104

(6 marks)

QUESTION TWO

a) Net Present Value (NPV) is the difference between the discounted cash inflows and the discounted cash outflows. The Internal Rate of Return is the rate that will make the NPV zero.

(4 marks)

(i) Cal	culation of NPV Ma	chine I	
Year	Discount Factor	Net Cash Flow	Present Value
	@10%		
0	1.0000	-10,000.00	-10,000.00
1	0.9091	5,000.00	4,545.50
2	0.8264	4,000.00	3,305.60
3	0.7513	2,000.00	1,502.60
4	0.6830	2,000.00	1,366.00
NPV			719.70

b)

Calculation of NPV Machine II

Year	Discount	Net Cash Flow	Present Value	
	Factor @10%			
0	1.0000	-10,000.00	-10,000.00	
1	0.9091	2,000.00	1,818.20	
2	0.8264	3,000.00	2,479.20	
3	0.7513	5,000.00	3,756.50	
4	0.6830	4,000.00	2,732.00	
NPV			785.90	

Machine II earn more returns than Machine I and So Machine II should be purchased.

(8 marks)

(ii) As Machine II has larger expected net inclement cash flow than Machine I in year 3 and year 4, it suffices to determine and compare the internal rate of returns for years 1,2 and 3 as follows:

Machine I : NPV @ y% is given by

$$-4000 + \frac{5000}{1+y} + \frac{4000}{(1+y)^2} = 0 \text{ implies } (1+y)^2 - 1.25(1+y) - 1 = 0$$
Therefore we have $(1+y) = \frac{1.25 \pm \sqrt{1.25^2 - 4(-1)}}{2} = \frac{1.25 \pm 2.3585}{2}$
which gives $(1+y) = 1.80425 \Rightarrow y = 0.8043$
Machine II : NPV @ y% is given by
 $-2,000 + \frac{2000}{1+y} + \frac{3000}{(1+y)^2} = 0$ implies $(1+y)^2 - (1+y) - 1.5 = 0$
Therefore, we have $(1+y) = \frac{1 \pm \sqrt{1+6}}{2} = \frac{1+2.6457}{2}$
which gives $(1+y) = \frac{3.6457}{2} = 1.82285 \Rightarrow y = 0.8229$

(6 marks)

(iii) Machine II earn more returns than Machine I and So Machine II should be purchased. (2 marks)



(8 marks)

b)				
Decision Node	Options	Expected Cost	Decision	
1	Organize	0.8x18,500+0.2x3,500	Organize	
	Exhibition for 7	=15,500	exhibition for 7	
	days	17,500	days	
2	Organize	0.3x15,500+0.7x2500	Do not exhibit	
	Exhibition for 5	=6,400	for 5 days	
	days	15,000		

(6 marks)

c) The Business Manager's optimal strategy is to exhibit the art works for 5 days and if no sales are made then extend the exhibition for another 2 days.

(6 marks)

a)



b)

i. Let X be the life span of the electronic components. Thus X ~ N(μ , σ^2) $P(x < 3500) = P(z < 3500 - \mu) = 0.01$ σ^2 $Z_{0.01} = -2.33$ And $z = 3500 - \mu$ σ Thus $-2.33 = 3500 - \mu$ σ ie $\mu - 2.33\sigma = 3500 \longrightarrow (i)$ $P(x>5500) = P(z>5500-\mu) = 0.025$ σ^2 $Z_{0.025}$ = 1.96 and Z = 5500 – μ σ Thus $1.96 = 5500 - \mu$ σ ie $\mu + 1.96\sigma = 5500 \longrightarrow$ (ii) (4 marks) ii. Solving equations (i) and (ii) simultaneously: $\mu - 2.33\sigma = 3500$ (i) $\mu + 1.96\sigma = 5500 \longrightarrow (ii)$ $\sigma = 2000 = 466.2$ hours 4.29 and substituting σ =466.2 in (i): μ = 3500+2.33(466.2) $\mu = 4586.25$ hours (4 marks) iii. P(4120.05<X<5052.45)=P(4120.05-4586.25<Z<5052.45-4586.25) 466.2 466.2 P(-1<Z<1)=2(0.34)=0.68 (4 marks) P(X < 4000) = P(Z < 4000 - 4586.25)iv. 466.2 P(Z < -1.2575) = P(Z < -1.26) = P(Z > 1.26)0.5-P(0<Z<1.26)= 0.5-0.3962 =0.1038 Under the warranty, the company can expect to replace about 10.38% of the components. (4 marks)

QUESTION FIVE

a)

i) Moving Average method is averaging observations within a period and using it as an estimate of the mid-point of the period of average.

(2 marks)

ii) Exponential Smoothing is a moving average system in which greater weights are attached to the most recent data. (2 marks)

(b)

x	У	5-Point	$\alpha = 0.5$	
		M.A		
1	34			
2	36		35.00	
3	24	32.0	29.50	
4	25	31.8	27.25	
5	41	31.8	34.13	
6	33	31.4	33.57	
7	34	31.8	33.79	
8	24	32.2	28.90	
9	23	32.6	25.95	
10	43	33.2	34.48	
11	35	33.4	34.74	
12	37	29.8	35.61	
13	25	30.6	30.31	
14	25		22.35	
15	47		34.68	

iii) The Graph of the $\alpha = 0.5$ exponential estimates are more closer to the actual observations.

(iv)



(v) The weighted moving average is approximating the actual number of enquiries better than the 5 period moving average values.

(16 marks)

QUESTION SIX

a)

i. **Collectively exhaustive events –** A set of events is jointly or collectively exhaustive if at least one of the events must occur. (2 marks)

ii. Complement of an event - The complement of an event is the event not occurring. Thus, the complement of Event A is Event A not occurring. (The probability that Event A will not occur is denoted by P(A')). (2 marks)

b)

- i) There are 171 students who chose boxing as their favourite sport and a total of 545 students levelicipated in the survey P(Boxing fan) =171/545 (2 marks)
 ii) A total of 155 students are in Level 1 P(selecting Level 1 student)= 155/545=31/109 or 0.284 (2 marks)
 iii) 70 students prefer hockey and a total of 210 students are in Level 2 P(Hockey fan/Level2 student)=70/210=1/3 or 0.333
- (3 marks) iv) A total of (84+70) 154 students prefer football or hockey P(Football or Hockey fan/Level 2 student) = 154/210=11/15 or 0.733
- v) A total of 211 students prefer football and 68 of them are in Level 1
 P(Level 1 student/Football fan)=68/211 or 0.322
- (3 marks) vi) A total of 180 students are in Level 3 and (74+47+59) 121 of them prefer football, boxing or hockey P(Football, Boxing or hockey fan/Level 3)=180/180 or 1

(3 marks) (Total: 20 marks)

QUESTION SEVEN

a) i) Maximise $P=15x_1+20x_2+14x_3$ Subject to: Fabrication: $5x_1+6x_2+4x_3 \le 210$ Materials: $10x_1+8x_2+5x_3 \le 200$ Assembly: $4x_1+2x_2+5x_3 \le 170$

$$\dot{x}_1 \ge 0, \dot{x}_2 \ge 0, x_3 \ge 0$$

Using s_1 , s_2 , s_3 as slack variables

 $5x_1+6x_2+4x_3+s_1=210$ $10x_1+8x_2+5x_3+s_2=200$ $4x_1+2x_2+5x_3+s_3=170$ $-15x_1-20x_2-14x_3+P=0$

ii)

(4 marks)



iii)

First iteration: $R_1 \rightarrow -6R_2+R_1$ $R_2 \rightarrow 1/8R_2$ $R_3 \rightarrow -2R_2+R_3$ $R_4 \rightarrow 20R_4+R_4$

	X ₁	X ₂	X ₃	S ₁	S ₂	S_3	RHS
S ₁	-2.5	0	0.25	1	-0.75	0	60
X ₂	1.25	1	0.625	0	0.125	0	25
S ₃	1.5	0	3.75	0	-0.25	1	120
Р	5	0	-1.5	0	2.5	0	500

In the first iteration, total profit is GH¢500.

(5 marks)

b)

i) **A sinking fund** is a fund containing money set aside or saved to pay off a debt or bond. A company that issues debt will need to pay that debt off in the future, and the sinking fund helps to soften the hardship of a large outlay of revenue.

(3 marks)

$$=\frac{50,000\times0.1}{\left[1-(1+0.1)^{-10}\right]}=\frac{50000}{6.144567}=GHS8,137.27$$

(4 marks)

(Total: 20 marks)

ii) Equal payment