SOLUTION 1

(a)

Q	Sales	Four Quarterly Moving Total	Four Quarterly Moving Average
1	200		
2	250		
3	254		
4	214	918	229.50
5	208	926	231.50
6	256	932	233.00
7	260	938	234.50
8	216	940	235.00
9	220	952	238.00
10	260	956	239.00
11	266	962	240.50
12	216	962	240.50
13	218	960	240.00
14	264	964	241.00

(b)

x	y	xy	x^2
4	229.50	918.00	16
5	231.50	1,157.50	25
6	233.00	1,398.00	36
7	234.50	1,641.50	49
8	235.00	1,880.00	64
9	238.00	2,142.00	81
10	239.00	2,390.00	100
11	240.50	2,645.50	121
12	240.50	2,886.00	144
13	240.00	3,120.00	169
14	241.00	3,374.00	196

$\Sigma x = 99 \quad \Sigma y = 2{,}602.5 \quad \Sigma xy = 23{,}552.5 \quad \Sigma x^2 = 1001$

$y = a + bx$ is the estimated trend line

$$b = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{n\,\Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{11 \times 23552.5 - 99 \times 2602.5}{11 \times 1001 - 99^2} = \frac{1430}{1210}$$

$$= 1.181818$$

(b) Average Seasonal Variance

Q_1	Q_2	Q_3	Q_4	
88.05	109.49	110.67	89.71	
89.71	109.85	91.75	92.99	
92.99	109.35	111.32	89.95	
90.34	108.87			
90.27	109.39	104.58	90.88	= 395.12

Adjusted	90.27 x 1.012351	109.39 x 1.0124	104.58 x 1.0124	90.88 x 1.0124	
	91.38	110.74	105.87	92.01	= 400

Seasonal forecast = Trend estimate and seasonal variances %

Year 1

Q_1 Forecast = 227.15 x 91.38%
= 207.57

Q_2 Forecast = 228.33 x 110.74%
= 252.85

Q_3 Forecast = 229.51 x 105.87%
= 242.98

Q_4 Forecast = 230.69 x 92.01%
= 212.26

b = 1.18

$$a = \bar{y} - b\bar{x}$$

$$= \frac{2602.5}{11} - 1.18\left(\frac{99}{11}\right)$$

$$= 236.59 - 10.62$$

$$= \underline{\underline{225.97}} \qquad y = 225.97 + 1.18x$$

Q	Sales	Trend line	Seasonal Variance	Seasonal Forecast
1	200	227.15	88.05	207.57
2	250	228.33	109.49	252.85
3	254	229.51	110.67	242.98
4	214	230.69	92.77	212.26
5	208	231.87	89.71	211.88
6	256	233.05	109.85	258.08
7	260	234.23	111.00	247.98
8	216	235.41	91.75	216.00
9	220	236.59	92.99	216.20
10	260	237.77	109.35	263.31
11	266	238.95	111.32	252.98
12	216	240.13	89.95	220.94
13	218	241.31	90.34	220.51
14	264	242.49	108.87	268.53

Estimated in (b) above forecast fairly well the actual sales the maximum absolute deviation is

$$\frac{12.02}{260} \times 100\% = 4.6\%$$

Alternative - SOLUTION 1

Q	Sales	Four Quarterly Moving Total	Four Quarterly Moving Average
1	200		
2	250	918	229.50
3	254	926	231.50
4	214	932	233.00
5	208	938	234.50
6	256	940	235.00
7	260	952	238.00
8	216	956	239.00
9	220	962	240.50
10	260	962	240.50
11	266	960	240.00
12	216	964	241.00
13	218		
14	264		

(b)

x	y	
2.5	229.50	
3.5	231.50	
4.5	233.00	$(\bar{x}_1, \bar{y}_1)$
5.5	234.50	
6.5	235.00	
7.5	238.00 →	$(\bar{x}_1, \bar{y})$
8.5	239.00	
9.5	240.50	
10.5	240.50	$(x_2, \bar{y}_2)$
11.5	240.00	
12.5	241.00	
$\Sigma x = 82.5$	$\Sigma y = 2,602.5$	

$$\bar{x} = \frac{82.5}{11}, \quad \bar{y} = \frac{2,602.5}{11}$$

$$= 7.5 \qquad = 236.59$$

$y = a + bx$ is the estimated trendline

$$b = \frac{\bar{y}2 - \bar{y}1}{\bar{2}_2 - x_1}$$

$$\bar{y}_1 = \frac{229.5 + 231.50 + 233.00 + 234.5 + 235.00}{5}$$

$\bar{y}_1 = 232.7$

$$\bar{x}_1 = \frac{2.5 + 3.5 + 4.5 + 5.5 + 6.5}{5}$$

$= 4.5$

$$\bar{y}_2 = \frac{239.00 + 240.50 + 240.50 + 240.00 + 241.00}{5}$$

$= 240.2$

$$\bar{x}_2 = \frac{8.5 + 9.5 + 10.5 + 11.5 + 12.5}{5}$$

$= 10.5$

$$b = \frac{240.2 - 232.7}{10.5 - 4.5}$$

$$= \frac{7.5}{6}$$

$= 1.25$

:. The trendline is given by

$y - \bar{y} = b (x - \bar{x})$

$y - 236.59 = 1.25 (x - 7.5)$

$y = 227.22 + 1.25x$

Q	Sales	Trendline	Seasonal Variation	Seasonal Forecast
1	200	228.47	87.54	202.27
2	250	229.72	108.83	249.66
3	254	230.97	109.97	255.71
4	214	232.22	92.15	210.83
5	208	233.47	89.09	209.44
6	256	234.72	109.07	255.09
7	260	235.97	110.18	261.24
8	216	237.22	91.05	215.63
9	220	238.47	92.25	213.93
10	260	239.72	108.46	260.53
11	266	240.97	110.39	266.78
12	216	242.22	89.18	220.18
13	218	243.47	89.54	218.41
14	264	244.72	107.88	265.40

Average Seasonal Variation

	Q_1	Q_2	Q_3	Q_4
	87.54	108.83	109.97	92.15
	89.09	109.07	110.18	91.05
	92.25	108.46	110.39	89.18
	89.54	107.88		
Average	89.61	108.56	110.58	90.79 = 399.54

Adjusted $89.61 \times \frac{400}{399.54}$ $\quad 108.56 \times \frac{400}{399.54}$ $\quad 110.58 \times \frac{400}{339.5}$ $\quad 90.79 \times \frac{400}{399.54}$

= 89.71 = 108.68 = 110.71 = 90.90 = 400

Seasonal Forecast = Trend estimate x seasonal variation %

Year 1

Q_1 Forecast = 228.47 x 89.71% = 202.27

Q_2 Forecast = 229.72 x 108.68% = 249.66

Q_3 Forecast = 230.97 x 110.71% = 255.71

Q_4 Forecast = 232.22 x 90.79% = 210.83

(b) Average Seasonal Variation

Q_1	Q_2	Q_3	Q_4
88.05	109.49	110.67	92.77
89.71	109.85	110.00	91.75
92.99	109.35	111.32	89.95
90.34	108.87		
90.27	109.39	111.00	91.49 = 402.15

Adjusted $90.27 \times \frac{400}{402.15}$ $\quad 109.39 \times \frac{400}{402.15}$ $\quad 111 \times \frac{400}{402.15}$ $\quad 90.88 \times \frac{400}{402.15}$

.

Seasonal forecast = Trend estimate and seasonal variation %

Year 1

Q_1 Forecast = 227.15 x 89.71% = 203.78

Q_2 Forecast = 228.33 x 108.80% = 248.42

Q_3 Forecast = 229.51 x 110.41% = 253.40

Q_4 Forecast = 230.69 x 91% = 209.93

Q	Sales	Trendline	Seasonal Variation	Seasonal Forecast
1	200	227.15	88.05	203.78
2	250	228.33	109.49	248.42
3	254	229.51	110.67	253.40
4	214	230.69	92.77	209.93
5	208	231.87	89.71	208.20
6	256	233.05	109.85	253.56
7	260	234.23	111.00	258.61
8	216	235.41	91.75	214.22
9	220	236.59	92.99	212.43
10	260	237.77	109.35	258.69
11	266	238.95	111.32	263.82
12	216	240.13	89.95	218.51
13	218	241.31	90.34	218.49
14	264	242.49	108.87	263.82

Estimated in (c) above forecast fairly well the actual sales. The maximum absolute deviation is $\frac{6.07}{220} \times 100\% = 3\%$

SOLUTION 2

(i) $P = \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix} \begin{matrix} A \\ B \end{matrix}$

(ii) $P^2 = \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix} \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix} = \begin{pmatrix} 0.66 & 0.34 \\ 0.17 & 0.83 \end{pmatrix}$

the required probability is 0.34

(iii) $P^3 = \begin{pmatrix} 0.66 & 0.34 \\ 0.17 & 0.83 \end{pmatrix} \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix}$

$= \begin{pmatrix} 0.562 & 0.438 \\ 0.219 & 0.781 \end{pmatrix}$

the required probability is 0.219

(iv) $(0.7 \quad 0.3) \begin{pmatrix} 0.438 \\ 0.781 \end{pmatrix} = 0.5409$

(v) Let q1 and q2 be the steady state probability.

$(q_1 \quad q_2) = (q_1 \quad q_2) \begin{pmatrix} 0.80 & 0.20 \\ 0.10 & 0.90 \end{pmatrix}$

$q_1 = 0.8q_1 + 0.10q_2$
$q_2 = 0.20q_1 + 0.90q_2$
and $q_1 + q_2 = 1$

$80 \ -2q_1 + 0.1q_2 = 0$ ①

$Q1 + q2 = 1$ ②

multiply ② by 0.2

$0.2q_1 + 0.2q_2 = 0.2$ ③

Add ① and ③ $0.3q_2 = 0.2$
$q_2 = 2/3$

put $q_2 = 2/3$ in eqn. ② we
have $q_1 = 1 - 2/3 = \underline{\underline{1/3}}$

Hence, after a long time, there is a 1/3 probability that a given customer shall buy lipstick A, and 2/3 that a customer shall buy lipstick B.

SOLUTION 3

A) $1 = r/m = \underline{0.036} = 0.003$, PMT = 100

i) The 1st payment of GHC100 will be made 1 month from now and its present value is:

$$PV = \frac{PMT}{(1+i)^n} = \frac{100}{1 + 0.003} = 100(1.003)^{-1}$$

$$= GHC99.70$$

<u>2 months</u>

$$PV = \frac{100}{(1+i)^2} = \frac{100}{(1.003)^2} = GHC99.40$$

<u>3 months</u>

$$PV = \frac{100}{(1+i)^3} = \frac{100}{(1.003)^3} = GHC99.10$$

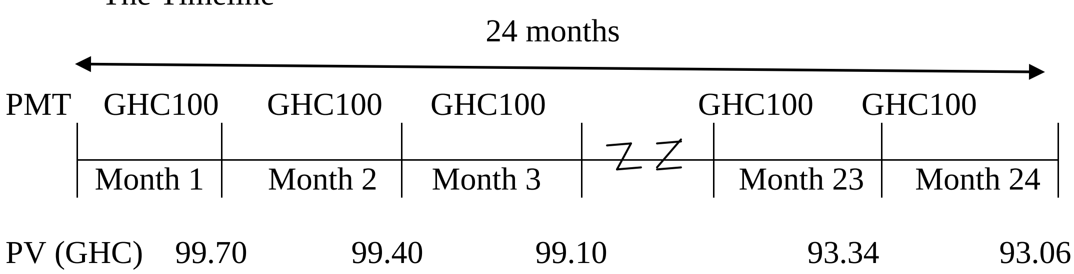
<u>23 months</u>

$$PV = \frac{100}{(1+i)^{23}} = \frac{100}{(1.003)^{23}} = GHC93.34$$

<u>24 months</u>

$$PV = \frac{100}{(1+i)^{24}} = \frac{100}{(1.003)^{24}} = GHC93.06$$

The Timeline



ii) For the account to be zero in 2 years then the present value PV

$$PV = PMT\ \frac{[1 - (1 + i)^{-n}]}{(1 + i) - 1}$$

$$= 100\ \frac{1 - (1 + 0.003)^{-24}}{(1 + 0.003) - 1}$$

$$= 100\ \frac{1 - (1.003)^{-24}}{0.003}$$

$$= GHC2,312.29$$

Adepa must initially deposit GHC2,312.29 in his account.

B) FV = 6000 , PV = 5000

r = 0.06 , m = 12

Then using $FV = PV\left(1 + \frac{r}{m}\right)^{mt}$

we have

$$6000 = 5000\left(1 + \frac{0.06}{12}\right)^{12t}$$

$$\left(1 + \frac{0.60}{12}\right)^{12t} = \frac{6000}{5000} = 1.2$$

Taking the logs of both sides

$$\log\left(1 + \frac{0.06}{12}\right)^{12t} = \log 1.2$$

$$12t \log\left(1 + \frac{0.06}{12}\right) = \log 1.2$$

$$t = \frac{\log 1.2}{12 \log\left(1 + \frac{0.06}{12}\right)}$$

$$t = \frac{\log 1.2}{12 \log (1.005)}$$

$t = 3.046$ years 3 years 1 month

:. It will take 3 years for Adepa's investment to grow from GHC5,000 to GHC6,000.

SOLUTION 4

a) The required matrix is $T = \begin{pmatrix} 0.6 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{pmatrix}$

b) The initial distribution vector is given by

$$X_0 = \begin{pmatrix} 0.80 \\ 0.15 \\ 0.05 \end{pmatrix}$$

Let X1denote the vector distribution after one observation. Then

$$X_1 = TX_0$$

$$X_1 = \begin{pmatrix} 0.6 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{pmatrix} \begin{pmatrix} 0.80 \\ 0.15 \\ 0.05 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 0.555 \\ 0.300 \\ 0.145 \end{pmatrix}$$

That is after all the Buses have made one pickup and discharged:

55.5% of the buses will be in Zone I
30% will be in Zone II
14.5% will be in Zone III

c) Let X_2 denote the distribution vector after all the buses made two pickups and discharged. Then

$$X_2 = TX1$$

$$X_2 = \begin{pmatrix} 0.6 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{pmatrix} \begin{pmatrix} 0.555 \\ 0.300 \\ 0.145 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} 0.4965 \\ 0.3000 \\ 0.2035 \end{pmatrix}$$

That is after all the buses have made two pickups and discharged:

49.65% of the buses will be in zone I
30% will be in Zone II
20.35% will be in Zone III

d) Let $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be the steady- state distribution vector.

Then the condition TX = X is:

$$\begin{pmatrix} 0.6 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

The system of linear equations is

$$0.6x + 0.4y + 0.3z = x$$
$$0.3x + 0.3y + 0.3z = y$$
$$0.1x + 0.3y + 0.4z = z$$

Condition for Long-run

$$x + y + z = 1$$

we have

$$4x - 4y - 3z = 0$$
$$3x - 7y + 3z = 0$$
$$x + 3y - 6z = 0$$
$$x + y + z = 1$$

Then

$$x -= \frac{4y + 3z}{4}$$
$$-16y + 21z = 0$$
$$8y + 7z = 4$$

$$z = \frac{8}{35} = 0.23$$
$$y = \frac{3}{10} = 0.30$$
$$\therefore x = 0.47$$

Solving: $x = \frac{33}{70}$ or 0.47, $y = \frac{3}{10}$ or 0.30, $z = 0.33$

47% of the buses will be in zone I
30% will be in Zone II
23 % will be in Zone III

SOLUTION 5

(a) (i) A turning point is a point on a graph where the gradient is zero.

(ii) 1 $((x) = 10 - 4x^3 + 3x^4$

$\frac{dc}{dx} = -12x2 + 12x3 = 0$

$- 12x^2 (1 - x) = 0$

⇨ $X = 0$ or $x = 1$

when $x = 0$, $((0) = 10$

$x = 1$, $((1) = 10 - 4 + 3$

$= 9$

So (0,10) and (1,9) are turning points.

Now $\frac{d^2c}{dx^2} = - 24x + 36x^2$

At (0,10), $\frac{d^2c}{dx^2} = -24(0) + 36(0) = 0$

⇨ (0,10) is a point of inflection

At (1,9) $\frac{d^2c}{dx^2} = -24 + 36 = 12 > 0$

⇨ (1,9) is a minimum point.

(b) AR = Average Revenue
MR = Marginal Revenue
AC = Average Cost
MC = Marginal Cost

(i) $AR = P = 15 - 2x$

$MR = \frac{dTR}{dx}$, where $TR = x(15 - 2x)$

$= 15 - 4x$

$MC = \frac{dc}{dx} = 2x + 2$

$AC = \frac{c}{x} = \frac{x^2 + 2x}{x} = x + 2$

(ii) For maximum profit

$$MC = MR \Rightarrow 15 - 4x = 2x + 2$$
$$6x = 13$$
$$x = 2\tfrac{1}{6}$$

Profit
$$\Pi(x) = x(15 - 2x) - (x^2 + 2x)$$
$$= 15x - 2x^2 - x^2 - 2x$$
$$= 13x - 3x^2$$

$$\Pi\left(\frac{13}{6}\right) = 13 \times \frac{13}{6} - 3\left(\frac{13}{6}\right)^2$$

$$= \frac{13}{6} \quad 13 - \left(\frac{3}{6} \times 13\right)$$

$$= \frac{13 \times 13}{6} \left(1 - \frac{3}{6}\right)$$

$$= \frac{13 \times 13}{6} \times \frac{1}{2}$$

$$= \frac{169}{12}$$

$$= \underline{\underline{14.08}}$$

(iii) - Graph

SOLUTION 6

(a) Distance is the explanatory variable. Time is the response variable.

(b) Graph and ((i).

(c) Table

X	Y	XY	X^2	Y^2
65	2	130	4225	4
137	2.5	342.5	18769	6.25
257	6	1542	66.49	36
84	2.25	189	7056	5.0625
105	3	315	11025	9
346	7.75	2681.5	119716	60.0625
124	3.25	403	15376	10.5625
157	3.75	588.75	24649	14.0625
201	4.5	904.5	40401	20.25
98	2.75	269.5	9604	7.5625
132	3.5	462	17424	12.25
1706	41.25	7827.75	334294	185.0625

(ii) $b = \frac{11\ (7827.75\ -\ (1706)\ (41.25)}{11\ (334294)\ -\ (1706)^2}$

$= \frac{86105.25\ -\ 70372.5}{3677234\ -\ 2910436}$

$= \frac{15732.75}{766798}$

$b = 0.025$

$a = \frac{41.25}{11} - (0.025)\ \frac{1706}{11}$

$= 3.75 - 3.182$

$a = 0.568$

hence $Y = 0.568 + 0.025x$

(iii) The Intercept "a" is a fixed time (just over half an hour) added onto each journey: this might be the time taken to load and unload vehicle.

- Gradient 'b' is the average time taken to travel a km on the salesman journeys.

$$r = \frac{11(7827.75) - (1706)(41.25)}{\sqrt{[11(334294) - (1706)^2][11(185.0625) - (41.25)^2]}}$$

$$r = \frac{86105.25 - 70372.5}{\sqrt{(766798)(334.125)}}$$

$$r = \frac{15732.75}{16{,}006.44813} = 0.9829$$

$$r = 0.9829$$

SOLUTION 7

Assistants	x	y	xy	x^2	y-23.78	$y-23.78^2$
1	0.4	16	6.4	0.16	-7.78	60.53
2	0.8	12	9.6	0.64	-11.78	38.77
3	0.8	20	16.0	0.64	-3.78	14.29
4	1.2	16	19.2	1.44	-7.78	60.53
5	1.4	34	47.6	1.96	10.22	104.45
6	1.8	30	54.0	3.24	6.22	38.69
7	2.2	26	57.2	4.84	2.22	4.93
8	2.6	22	57.2	6.76	-1.78	3.16
9	3.0	38	114.0	9.00	14.22	202.21
10	$\Sigma x=14.2$	$\Sigma y=214$	$\Sigma xy=381.2$	$\Sigma x^2=28.68$		

i. $y = a + bx$

$$b = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{9(381.2) - (14.2)(214)}{9(28.68) - (14.2)^2}$$

$$b = \frac{3430.8 - 3038.8}{258.12 - 201.64}$$

$$b = \frac{392}{56.48} = \mathbf{6.94}$$

a= y-(6.94) (x)

$$= \frac{214}{9} - (6.94)\frac{(14.2)}{9}$$

$= 23.78 - (6.94)(1.558)$

$= 23.78 - 10.81$

a = **12.97**

ii. y=12.97 + 6.94x

if x = 2.5; then y = 12.97 + 6.94(2.5)

= 12.97 + 17.35

= **30.32**

iii. y=12.97 + 6.94x

if x = 1.2; then y = 12.97 + 6.94(1.2)

= 12.97 + 8.328

= **21.298**

iv. the standard error of the mean would be

$$\acute{S} = \sqrt{\frac{\Sigma(y-\bar{y})^2}{N-1}}$$

$$= \sqrt{\frac{425.34}{9-1}}$$

$$= \sqrt{53.17}$$

= **7.29**

v. the expected change is given by $\bar{y}$ = a +bx as b = **6.94**