SOLUTION 1

(a) (i) Marginal Cost $MC = 6Q^2 - 6Q + 400$

Total Cost $TC = \int McdQ$

$= \int (6Q2 - 6Q + 400)\, dQ$

$= 2Q^3 - 3Q^2 + 400Q + C$

Where C is the constant of integration
(ie C is fixed cost in GHS'000)

When Q = 10, TC = 10700
ie $10700 = 2(10)3 - 3(10)^2 + 400(10) + C$

$= 5000$

Hence the fixed cost is GHS5,000,000

(ii) Total cost : $TC = 2Q^3 - 3Q2 + 400 + 5000$

Unit Price : $P = 4000 - 33Q$

Total Revenue : $TR = P\,Q$

$= 4000Q - 33Q^2$

$TI = TR - TC$

$= 4000Q - 33Q^2 - 2Q^3 + 3Q^2 - 4000 - 5000$

$$\frac{d\Pi}{dQ} = 6Q^2 - 60Q + 3600$$

For maximum profit, $\frac{d\Pi}{dQ} = 0$

ie $-6Q^2 - 60Q + 3600 = 0$

or $Q^2 + 10Q - 600 = 0$

For $ax^2 + bx + C = 0;$

$$x = \frac{-b = \sqrt{6^2 - 4ac}}{2a}$$

where $x = Q$, $a = 1$, $b = 10$, $c = -600$

$$\therefore\ Q = \frac{-10 \pm \sqrt{10^2 - 4(1)(-600)}}{2(1)}$$

$$= \frac{10 \pm 50}{2}$$

$= -30$ or 20

Hence an output of 2000 will maximize profit.

NB Accept Marginal cost = Marginal revenue approach.

(iii) The point elasticity of demand is:

$$PED = \frac{P}{Q} \Big/ \frac{dP}{dQ}$$

where $Q = 20$

$P = 4000 - 33\,(20) = 3340$

$$\frac{dP}{dQ} = -33$$

$$\therefore\ PED = \frac{3340}{20} \Big/ (-33)$$

$$= \quad 5.06$$

(b) Total tax: $Ta = 1548Q$

Profit after tax: $\Pi a = \Pi - Ta$

$$= -2Q^3 - 30Q^2 + 2052Q - 5000$$

$$\frac{d\Pi a}{dQ} = -6Q^2 + 2052$$

At maximum Πa, $\frac{d\Pi a}{dQ} = 0$

ie $-6Q^2 - 60Q + 2052 = 0$

or $Q^2 + 10Q - 342 = 0$

$$\therefore\ Q = \frac{-10 \pm \sqrt{102 - 4(1)(-342)}}{2(1)}$$

$$= 14.167 \text{ or } -24.167$$

Choose $Q = 14.16$ since $Q \geq 0$

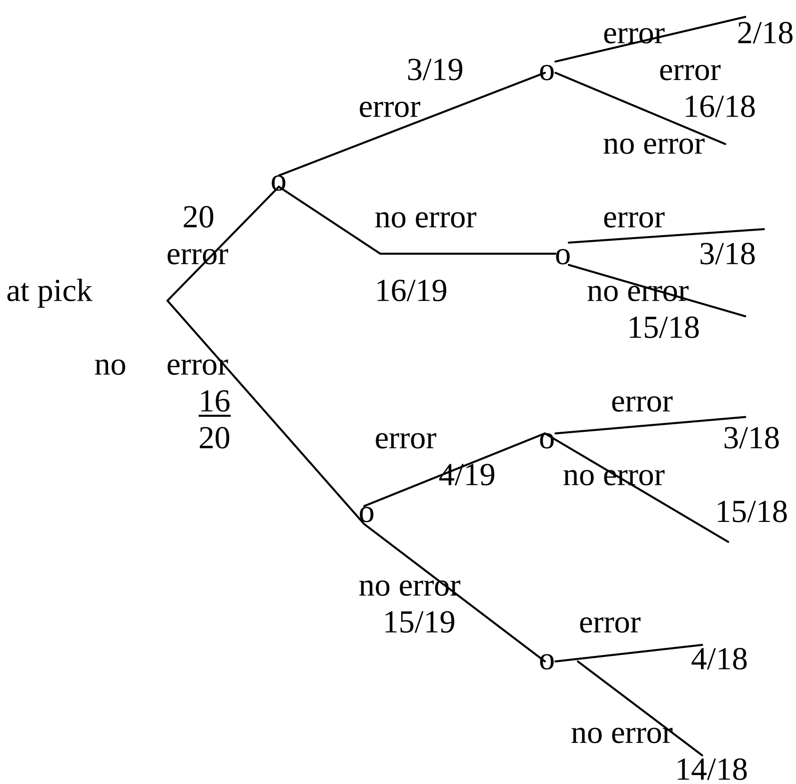
Item	Before Tax	After Tax	Remarks
Output	2000	1416	Decrease
Price (P)	3340000	3532720	Increase
Profit (Π)	39000000	12362833	Decrease

SOLUTION 2

(a) <u>Mutually Exclusive events</u>: Two events are mutually exclusive if the occurrence of either event <u>excluded the possibility</u> of the occurrence of the other event, that is, either one event or the other event but not both can occur.

<u>Independent Events</u>: Two events are independent if the occurrence or own occurrence f one event <u>has no influence</u> on the occurrence or non occurrence of the other event.

(b) <u>Probabilities</u>



$\frac{4}{20} \times \frac{3}{19} \times \frac{2}{18}$ (EEE)

$\frac{4}{20} \times \frac{43}{19} \times \frac{16}{18}$ (EEE)

$\frac{4}{20} \times \frac{16}{19} \times \frac{3}{18}$ (EĒE)

$\frac{4}{20} \times \frac{16}{19} \times \frac{15}{18}$ (EĒĒ)

$\frac{16}{20} \times \frac{4}{19} \times \frac{3}{18}$ (ĒEE)

$\frac{16}{20} \times \frac{4}{19} \times \frac{15}{18}$ (ĒEĒ)

$\frac{16}{20} \times \frac{15}{19} \times \frac{4}{18}$ (ĒĒE)

$\frac{16}{20} \times \frac{15}{19} \times \frac{14}{18}$ (ĒĒĒ)

(i) P (EEE) $= \frac{4}{20} \times \frac{3}{19} \times \frac{2}{18} = \frac{1}{285}$

(ii) P (one in error only) $= \left(\frac{4}{20} \times \frac{16}{19} \times \frac{15}{18}\right) + \left(\frac{16}{20} \times \frac{4}{19} \times \frac{5}{18}\right) + \left(\frac{16}{20} \times \frac{15}{19} \times \frac{4}{18}\right)$

$= 8/19$

(iii) P (two in error only) $= \left(\frac{4}{20} \times \frac{3}{19} \times \frac{16}{18}\right) + \left(\frac{16}{20} \times \frac{4}{19} \times \frac{3}{18}\right) + \left(\frac{4}{20} \times \frac{16}{19} \times \frac{3}{18}\right)$

$= 8/95$

(iv) P (non in error) $= \frac{16}{20} \times \frac{15}{19} \times \frac{4}{18}$

$= 28/57$

(c) Assume we have the same number of erroneous and good invoice
Let E be event "erroneous" increase
G be event "Good" invoices
C be correctly picked invoices

$P(C) = P(C \cap E) + P(G \cap C)$

$= P(C/E).P(E) + P(C/G).P(G)$

$P(C/E) = 0.6 \qquad P(C/G) = 0.7$

$P(C) = 0.5 \qquad P(G) = 0.5$

$P(C) = 0.6 \times 0.5 + 0.7 \times 0.5$

$= 0.65$

(i) P (All three are erroneous) $= \binom{3}{3}(0.35)^3 \; (0.65)^{3-3}$

$= (0.35)^3$

(ii) P (Only one is erroneous) $= \binom{3}{1}(0.35)^1 \; (0.65)^{3-1}$

$= 0.444$

(iii) $\overset{\frown}{\coprod}$ (Only two are erroneous) $= \begin{pmatrix}3\\2\end{pmatrix}(0.35)^2 \ (0.65)^{3-2}$

$= 0.2389$

(iv) $\overset{\frown}{\coprod}$ (None is erroneous) $= \begin{pmatrix}3\\0\end{pmatrix}(0.35)^0 \ (0.65)^{3-0}$

$= 0.2746$

SOLUTION 3

(a)

Year	Month	Data value	Cumulative Total	Total for year ending at stated month
2009	January	15	15	248
	February	20	35	256
	March	24	59	265
	April	30	89	276
	May	33	122	288
	June	36	158	299
	July	40	198	310
	August	39	237	319
	September	35	272	326
	October	30	302	331
	November	20	322	332
	December	15	337	337

Graph for Z- Chart

Year	Month	Production levels('000) (Y)	12MT	24MY	Centred 12 MA (T)
2008	January	10			
	February	12			
	March	15			
	April	19			
	May	21			
	June	25			
	July	29	243		
	August	30	248	491	20.5
	September	2	256	504	21.0
	October	25	265	521	21.7
	November	19	276	541	22.5
	December	10	288	564	23.5
2009	January	15	299	587	24.5
	February	20	310	609	25.4
	March	24	319	629	26.2
	April	30	326	645	26.9
	May	33	331	657	27.4
	June	36	332	663	27.6
	July	40			
	August	39			
	September	35			
	October	30			
	November	20			
	December	15			

$$\text{Incremental trend} = \frac{27.6 - 20.5}{18 - 7}$$

$$= 0.71$$

Forecast for 2010:

Month	Trend (T)	S	Forecast ($\hat{Y}$)
January	27.6 + 8 x 0.71 = 33.28	-10.4	22.88 $\simeq$ 23
February	27.6 + 9 x 0.71 = 33.99	-6.2	27.79 $\simeq$ 28

SOLUTION 4

(a) Let the quantity of goods sold be Q, selling price of the goods be P, and the cost of the goods be C

Then

$$Q = \begin{pmatrix} 1000 \\ 600 \\ 1200 \end{pmatrix}, \quad P = \begin{pmatrix} 4.5 \\ 6.0 \\ 5.0 \end{pmatrix}, \quad C = \begin{pmatrix} 3.8 \\ 4.2 \\ 3.2 \end{pmatrix}$$

$$\text{Total revenue, } TR = P^TQ = (1.5 \quad 6.0 \quad 5.0) \begin{pmatrix} 1000 \\ 600 \\ 1200 \end{pmatrix}$$

$$= 14{,}100$$

$$\text{Total cost, } TC = CTQ = (3.8 \quad 4.2 \quad 3.2) \begin{pmatrix} 1000 \\ 600 \\ 1200 \end{pmatrix}$$

$$= 10{,}160$$

Profit = Total Revenue – Total Cost

= 14,100 - 10,160

= 3,940

Profit for the week is = GHS3,940

(b) $8P_1 + 2P_2 = 470$

$P_1 + 5P_2 = 415$

$$\text{Let } A = \begin{pmatrix} 8 & 2 \\ 1 & 5 \end{pmatrix}$$

$\det A = (8 \times 5) - (1)(2) = 40 - 2 = 38$

$$\text{adj } A = \begin{pmatrix} 5 & -2 \\ -1 & 8 \end{pmatrix}$$

$$\therefore A\text{-}1 = \frac{\text{adj } A}{\det A} = \underline{1} \begin{pmatrix} 5 & -2 \\ -1 & 8 \end{pmatrix}$$

Matrix equation for equilibrium

$$\begin{pmatrix} 8 & 2 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 470 \\ 415 \end{pmatrix}$$

Remultiply this equation by A -1 to obtain

$$\begin{pmatrix} P1 \\ P2 \end{pmatrix} = \frac{1}{38} \begin{pmatrix} 5 & -2 \\ -1 & 8 \end{pmatrix} \begin{pmatrix} 470 \\ 415 \end{pmatrix}$$

$\therefore P_1 = 40$ and $P_2 = 75$

(c) The augmented matrix of the system of equations is given by

$$\left(\begin{array}{ccc|c} 6 & 2 & 5 & 73 \\ 7 & -3 & 1 & -1 \\ 1 & 8 & -9 & -9 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{5}{6} & \frac{73}{6} \\ 7 & -3 & 1 & -1 \\ 4 & 8 & -9 & -9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{5}{6} & \frac{76}{6} \\ 0 & \frac{-16}{3} & \frac{-29}{6} & \frac{-517}{6} \\ 0 & \frac{20}{3} & \frac{74}{6} & \frac{-173}{3} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{5}{6} & \frac{73}{6} \\ 0 & 1 & \frac{29}{32} & \frac{517}{32} \\ 0 & \frac{20}{3} & \frac{-74}{6} & \frac{-173}{3} \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{51}{96} & \frac{651}{96} \\ 0 & 1 & \frac{29}{32} & \frac{517}{32} \\ 0 & 0 & \frac{-441}{24} & \frac{-3969}{24} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{51}{96} & \frac{651}{96} \\ 0 & 1 & \frac{29}{32} & \frac{517}{32} \\ 0 & 0 & 1 & 9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 9 \end{array}\right)$$

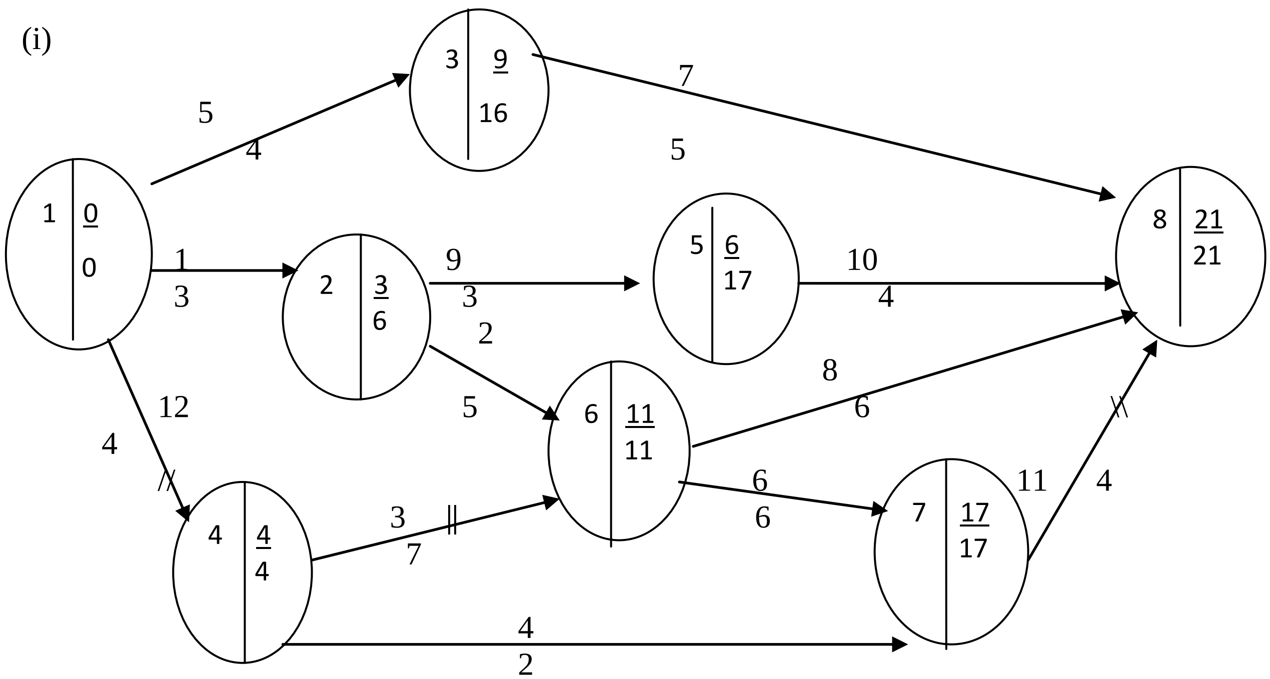
$\therefore X_1 = 2; \quad X_2 = 8; \quad X_3 = 9$

SOLUTION 5

(a) A Chain Float is the spare time associate with a chain of non-critical activities.

(b) Free Float is the amount of time an activity can be delayed without affecting the commencement of a subsequent activity of its earliest start time, but may affect float of a previous activity. An independent float is the delay when all preceding activities are completed as late as possible and all succeeding activities completed as early as possible.
Independent float = (EET – LST - D)
Free float = (EET - EST - D)

(c) (i)



(ii) The critical path from the diagram is 12 - 3 - 6 - 11
The diagram of the project is 21 days

(iii)

<u>Activity</u>	Total float <u>(LET – EST –D)</u>	FREE Float (EET -EPT – D)	Independent Float <u>(EET – LST – D)</u>
1	(6 – 0 -3) 3	(3 – 0 – 3) 0	(6 – 0 -3) 3
2	(11 – 3 – 5) 3	(11 – 0 – 5) 6	(11 – 6 – 3) 2
3	(11 – 4 7) 0	(11 – 4 – 7) 0	(11 – 4 – 7) 0
4	(17 – 4 – 2) 11	(17 – 4 – 2) 11	(17 – 4 -2) 11
5	7	0	0
6	0	0	0
7	7	7	4
8	4	4	8
9	11	0	0
10	11	11	0
11	0	0	
12	0	0	

(v) Activities that have Total Float = free float
Independent Float = 0 are Critical path activities

SOLUTION 6

(a)

Year	Qtr	Revenue	Sun in qtrs	Sum of … qtrs	Trend	Actual-Trend	Actual Yield
2007	1	37					
	2	58	211				
	3	67	212	423	52.875	14.125	4.749
	4	49	213	425	53.125	-4.125	11.879
2008	1	38	214	427	53.375	-15.375	2.472
	2	59	215	429	53.625	5.375	10.977
	3	38	217	432	54.000	14.000	4.857
	4	50	218	435	54.375	-14.375	11.429
2009	1	40	220	438	54.750	-4.750	8.421
	2	60	221	441	55.125	4.875	122.308
	3	70	223	444	55.500	14.500	4.828
	4	51	224	447	55.875	-4875	10.462
2010	1	42	226	450	56.250	-14.250	2.947
	2	61					
	3	72					

(b)

Year	Q1	Q2	23	Q4
2007			14.125	-4.125
2008	-15.375	5.375	14.000	-4375
2009	-14.750	4.875	14.500	-4875
2010	-14.250			
Total	-44.375	10.25	42.625	13.75
Average	-14.792	5.125	14.208	-4583
Adj	0.0105	0.0105	0.0105	-0.823
Seasonal Variation	-12.281	4.302	13.385	-5.406

(c) A reasonable trendline is drawn on the graph sheet.
From the trendline

4^{th} Quarter of 2010, trend = 57.35
1^{st} Quarter of 2011, trend = 57.70
2^{nd} Quarter of 2011, trend = 58.10
3^{rd} Quarter of 2011, trend = 58.45

$$\text{Gradient} = \frac{56 - 25 - 52.895}{11 - 1} = 0.3375$$

Estimated data value = Forecast trend value + appropriate seasonal variation value

4^{th} Quarter of 2010 estimated value = 57.35 – 5 = 52.35
1^{st} Quarter of 2011 estimated value = 57.70 – 11 = 46.70
2^{nd} Quarter of 2011 estimated value = 58.10 + 5 = 63.10
3^{rd} Quarter of 2011 estimated value = 58.45 + 14 = 72.45

SOLUTION 7

Let denote the occupancy rate of Hotel A. Then the average daily rate of occupancy is given by the expected value of X.

$$\begin{aligned} E(x) &= (0.80)(0.19) + (0.85)(0.22) + (0.90)(0.31) + (0.95)(0.23) + (-1.00)(0.05) \\ &= 0.152 + 0.187 + 0.279 + 0.2185 + 0.05 \\ &= 0.8865 \end{aligned}$$

Let Y denote the occupancy rate of the Hotel B. Then the average daily rate of occupancy is given by the expected value Y.

$$\begin{aligned} E(Y) &= (0.75)(0.35) + (0.80)(0.21) + (0.85)(0.18) + (0.90(0.15) + (0.95)(0.09) + (1.00)(0.02) \\ &= 0.2625 + 0.168 + 0.153 + 0.135 + 0.0855 + 0.02 \\ &= 0.8242 \end{aligned}$$

The average number of rooms occupied per day at the Hotel B is (0.8240 (60) = 49.4

The expected daily profit at Hotel A is given by

$$(46.1)(10) = 461 \text{ or GHS461}$$

The expected daily profit at Hotel B is given by

$$(49.4)(9) = 445 \text{ or GHS445}$$

From these results we conclude that the investors should purchase Hotel A which is expected to yield a higher daily profit.