SOLUTION 1

(a) Let Uniform amount annuity = X

Accumulate amount = F

Payment		x	x	x	x	x	x	x	x	x	x
Time	0	1	2	3	4	5	6	7	8	9	10

(b) $F = X \left[\frac{(1+i)^n - 1}{i} \right] = z \ Sn \ i$

$X = \frac{F}{\left[\frac{(i+i)^n}{I} \right.} = \frac{100{,}000}{\left. \frac{(1.07)^{10} - 1}{0.07} \right]}$

= GHS7,237.75

(c) X = 6000 , F = 100,000 , E = 7% n = ?

$F = X \left[\frac{(1+i)^n - 1}{i} \right]$

$100{,}000 = 6000 \left[\frac{(1 + 0 - 07)^n - 1}{0.07} \right]$

$\left[\frac{100}{6} \right] (0.07) = (1 + 0.07^n - 1$

$1.07^n = 1 + (7/6)$

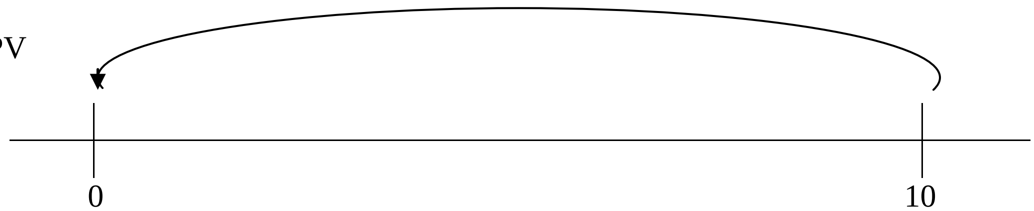
Taking ln of both sides

$\ln 1.07^n = \ln (1.1667)$

$n \ln 1.07 = \ln (1.1667)$

$\Rightarrow n = \frac{\ln (1.1667)}{\ln 1.07} = 11.43$ years

(d) PV



C = 7% , n = 10

One time investment = Present value (PV) of GHS100,000

$= FV\ (1 + i)^{-n}$

$= 100{,}000\ (1.07)^{-iv}$

= GHS50,834.93

or

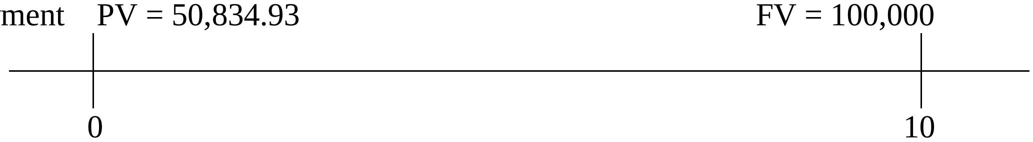
GHS50.835

or (CAO)

GHS50834.9

(e)

. Payment PV = 50,834.93 FV = 100,000



Time

SOLUTION 2

Let x =N of Servings of Vanilla Pudding

y = Number of Serving of Chocolate Pudding

The objective function sis:

Maximize: p = 10x + 7y

Table for the data

	Vanilla	Chocolate	Total Available
Sugar (teaspoons)	2	3	3,600
Water (fl oz)	25	15	22,500

LP problem is:

Maximize: p = 10x + 7y

Subject to: $2x + 3y \leq 3600$

$5x + 3y \leq 4500$ we divide $5x + 3y \leq 22500$ by 5

$x \leq 600$

$x \geq 0,\ y \leq 0$

Introducing the slack variable we have:

$$x + s = 600$$
$$2x + 3y + t = 3600$$
$$5x + 3y + u = 4500$$
$$-10z - 7y + p = 0$$

Using Simples method to solve the problem:

pivot

	x	y	s	t	u	p	
s	1	0	1	0	0	0	600
t	2	3	0	1	0	0	3600
u	3	3	0	0	1	0	4500
p	-10	-7	0	0	0	1	0

Pivot column

	x	y	s	t	u	p	
x	1	0	1	0	0	0	600
t	0	3	-2	1	0	0	2400 $R_2 - 2R_1$
u	0	3	-5	0	1	0	1500 $R_3 - 5R_1$
p	0	-7	10	0	0	1	600 $R_4 + 10R_1$

	x	y	s	t	u	p	
x	1	0	1	0	0	0	600
t	2	0	3	1	-1	0	900 $R_2 - R_3$
y	0	3	-5	0	1	0	1500
p	0	0	-5	0	7	3	0 $3R_4 + 7R_3$

	x	y	s	t	u	p	
x	3	0	0	-1	1	0	900 $3R_1 - R_2$
t	0	0	3	1	-1	0	900
y	0	9	0	5	-2	0	9000 $3R_3 + 5R_2$
p	0	0	0	5	16	9	90000 $3R_4 + 5R_2$

This solution is:

The maximum value of $p = \frac{90000}{9} = 10000 = \text{GHS}100$

Which occurs at $x = \frac{900}{3} = 300$

$y = \frac{9000}{9} = 1000$

Slack variables $s = \frac{900}{3} = 300$

$t = u = 0$

SOLUTION 3

(a) i. Transition matrix is a square matrix which gives the number or proportion of times that same process will change from one state to another in a defined period of time.

ii. 1.2 The sum of the proportions in each row (column) must add to one (i).

2. Transition matrix is square matrix.

3. The entries must be negative

(b) Given that $p = \begin{bmatrix} 1 & 7 \\ 0 & 4 \end{bmatrix}$ and $T = \begin{bmatrix} 2 & 4 \\ 2 & 0 \end{bmatrix}$

$$p^{-1} = \frac{1}{(4 \times 1) - (0 \times 7)} \begin{bmatrix} 4 & -7 \\ 0 & 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 & -7 \\ 0 & 1 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 2 & 4 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4+8 & 8+0 \\ 4+0 & 8+0 \end{bmatrix} = \begin{bmatrix} 12 & 8 \\ 6 & 8 \end{bmatrix}$$

$$\therefore K = T^2 P^{-1} = \frac{1}{4} \begin{bmatrix} 12 & 8 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} 4 & -7 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \quad \begin{matrix} 4 & -7 \\ 0 & 1 \end{matrix} = \begin{bmatrix} 12+0 & -21+2 \\ 4+0 & -7+2 \end{bmatrix}$$

$$K = \begin{bmatrix} 12 & -19 \\ 4 & -5 \end{bmatrix}$$

(c) Probability Transition Matrix for ABC Company is

i.

	Abay	Babay	Cabay
Abay	0.6	0.4	0.1
Babey	0.2	0.5	0.1
Cabay	0.2	0.1	0.8

ii. The initial market share vector is

Abay	40
Babay	40
Cabay	20

iii. New market share

$$\begin{bmatrix} 0.6 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 40 \\ 40 \\ 20 \end{bmatrix} = \begin{bmatrix} 24 + 16 + 2 \\ 8 + 20 + 2 \\ 8 + 4 + 16 \end{bmatrix} = \begin{bmatrix} 42 \\ 30 \\ 28 \end{bmatrix}$$

The new market share after wee one is 42:30:38 for Abay, Babay and Cabay respectively.

iv. New market share after week two is

$$\begin{bmatrix} 0.6 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 42 \\ 30 \\ 28 \end{bmatrix} = \begin{bmatrix} 25.2 + 12.0 + 2.8 \\ 8.4 + 15 + 2.8 \\ 8.4 + 3 + 22.4 \end{bmatrix} = \begin{bmatrix} 40.0 \\ 26.2 \\ 33.8 \end{bmatrix}$$

The new market share after week two is 40.0%, 26.2% and 33.8% for Abay, Babay and Cabay respectively.

v. Let A, B, C, denote equilibrium percentage market share
then A + B + C = 1

and
$$\begin{bmatrix} 0.6 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 0.6A + 0.4B + 0.1C &= A \\ 0.2A + 0.5B + 0.1C &= B \\ 0.2A + 0.1B + 0.8C &= C \\ A + B + C &= 1 \end{aligned}$$

<u>Solving</u> the above equations simultaneously

A = 5.56%
B = 7.41%
C = 87.03%

SOLUTION 4

a) In an activity-on-mode diagram the modes represent activities but in an arrows (lines) represent the activities.

b) (i)

Activity	Mean duration (days)	Standard deviation (days)
A	7.8	0.83
B	5	0.67
C	7.3	1.33
D	10	0.67
E	6	0.00
F	3.2	0.50
G	3	0.67
H	8	0.33
I	5.2	0.83

ii. The Network Diagram

Alternative Network Diagram

(iii) Average project duration: …… 31 8 days

Critical path is D – E – H – A

:. Variance of project duration: $\ldots^2 = 067^2 + 0.33^2 + 0.83^2$

$= 1.2467$

:. $\sigma = 1.1166$

Let X be the project duration (in days)

Then $X_2N\ (31.8, 1.116^2)$

$$P(X \leq 34) = P\left(\frac{x - 31.8}{1.1166} \leq \frac{34 - 31.8}{1.1166}\right)$$

$P(\leq 34) = P(Z \leq 1.97)$

From tables, $P(x \leq 34) = 9.756$

$\therefore$ Expected Bonus = 500 x 0.9756
= GH¢4878

$P(x > 34) = 1 - P(x \leq 34)$
$= 1 - 0.9756$
$= 0.0244$

$\therefore$ Expected Penalty = 10,000 x 0.0244
= GH¢244

SOLUTION 5

(a) Data Collection Methods

(i) Face to face interview (Primary data)
(ii) Mail questionnaire (Primary data)
(iii) Observation (Primary data)
(iv) Experimentation (Primary data)
(v) Documents/Report (Primary data)

(b) (i) <u>Frequency Distribution Table</u>

Marks (%)	Tally	Frequency	Class boundaries
20 – 29	~~////~~ ~~////~~ ///	13	19.5 – 29.5
30 – 39	~~////~~ ////	9	29.5 – 39.5
40 – 49	~~////~~	9-	39.5 – 49.5
50 – 59	////	4	49.5 – 59.5
60 – 69	~~////~~ ////	9	59.5 – 69.5
70 – 79	~~////~~ /	6	69.5 – 79.5
80 – 89	~~////~~ ~~////~~ //	12	79.5 – 89.5
90 - 99	~~////~~ ~~////~~	10	89.5 – 99.5
		70	

(iii) 1^{st} Quartile = 34.6

2^{nd} Quartile = 61.75

3^{rd} Quartile = 83.5

Modal Mark = 27.5

The Bowley's Coefficient of skewed is defined as

$$B_{CSK} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{83.5 + 34.6 - 2(61.75)}{83.5 - 34.6}$$

$$= \frac{-5.4}{48.9}$$

$$= -0.1104$$

Since B_{CSK} is less than zero, the distribution is negatively skewed.

SOLUTION 6

(a) The volume is normally distributed with mean M = 475 and Standard Deviation σ = 20
V~N (475, 20^2) , $Z = \frac{X - M}{\sigma}$

The probability that the volume is less than 482 L

(i) $Z = \frac{480 - 475}{20} = 0.25$
… (0.25) = 0.59871 from tables
P (V < 480) = 0.5987

The probability that the volume will be less than 480 L is0.5987

(ii) The probability that the volume is between 460 L and 490 L is

P (460 L V < 490)
$Z = \frac{460 - 475}{20} = -0.75$
$Z = \frac{490 - 475}{20} = 0.75$

φ (0.75) = 0.77337 from tables
P (460L V < 490) = 0.77337 – (1 – 0.77337)
:. The probability that the volume is between 460 L and 490 L is 0.54674

(iii) <u>To be provided</u>

$Z = \frac{490 - 475}{20} = \frac{15}{20} = 0.75$

P (Z20.75) = 0.500 – 0.2734 = 0.2266

(b) The probability that the volume is greater than 500 L

$Z = \frac{500 - 475}{20} = 1.25$

φ (1.25) = 0.89435

P (V > 500) = 1 – 0.89435
= 0.10565

The probability that the amount of liquid soap poured is greater than the capacity of the container and so over flows is 0.10565.

(c) If the probability of overflowing is 0.001. Then the probability of not overflowing is:
Then P = 1 – 0.001 = 0.999
From tables Z = 3.09
Therefore $3.09 = \frac{500 - N}{20}$

N = 500 – 3.09 x 20
M = 438.2

The engineer can adjust the mean value that is poured to 438.2 Litres.

SOLUTION 7

(a) The properties of linear correlation coefficient, r are:

(i) The value of r is always between .. and .. inclusive.
(ii) The value of r does not change if all values of either variable are correlated to a different scale.
(iii) The value of r is not affected by the interchange of the valuer or data of computation.
(iv) R measures the strength of linear relationship of two sets of data.

(b) (i) See graph

(ii) and (iii)

Output (x)	Level rank x	Dexterity (y)	vy	x^2	y^2	xy	d	d2
86	(9)	6	(6.5)	7396	36	516	+2.5	6.25
51	(2)	4	(3)	2601	16	204	-1	1
101	(13)	7	(8.5)	10201	49	707	+4.5	20.25
91	(11)	10	(15)	8281	100	910	-4	16
77	(8)	4	(3)	5929	16	308	-15	25
58	(4)	7	(8.5)	3364	49	406	-4.5	20.25
75	(7)	9	(13)	5625	81	675	-6	36
110	(15)	8	(10.5)	12100	64	880	+4.5	20.25
99	(12)	9	(13)	9801	81	891	-1	1
106	(14)	6	(6.5)	11236	36	636	+7.5	56.25
52	(3)	8	(10.5)	2704	64	416	-7.5	56.25
44	(1)	5	(5)	1936	25	220	-4	16
88	(10)	4	(3)	7744	16	352	+7	49
67	(6)	9	(13)	4489	81	603	-7	49
63	(5)	2	(1)	3969	4	126	+4	16
1168		98		97376	718	7850		388.5

Graph to be provided

$$r = \frac{n\,\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[n\,\Sigma x^2 - (\Sigma x)^2]\,[n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{(15)(7852) - (1168)(98)}{\sqrt{[(15)(97376) - (11687)^2]\,[(15)(718)^2]}}$$

$$= \frac{3286}{\sqrt{[1460640 - 1364224]\,10770 - 9604]}}$$

$$= \frac{3286}{\sqrt{(96416)(1166)}} = \frac{3286}{10602.9}$$

$$= 0.3099 = 0.31$$

$$R = 1 - \left(\frac{6\,\Sigma d^2}{n(n^2-1)}\right) - \left(\frac{t^3 - t}{12}\right) = \frac{1 - 6(\Sigma d2 + 110)}{n(n^2-1)}$$

$$= 1 - \frac{(6)(498.5)}{(15)(224)}$$

$$= 1 - \frac{2391}{3360} = 1 - 0.7114$$

$$= 0.109$$

(iv) From the two results notably

R = 0.3099 ; R = 0.1099

and the scatter diagram, r = +0.3099 is preferable. Reason being that the two sets of data are sparsely correlated and r is closer to unity.