ASSOCIATION OF ACCOUNTANCY BODIES IN WEST AFRICA

ACCOUNTING TECHNICIANS SCHEME, WEST AFRICA
PART II EXAMINATIONS – SEPTEMBER 2021

QUANTITATIVE ANALYSIS

Time Allowed: 3 hours

SECTION A: PART I MULTIPLE-CHOICE QUESTIONS (30 Marks)

ATTEMPT ALL QUESTIONS

Write ONLY the alphabet (A, B, C, D or E) that corresponds to the correct option in each of the following questions/statements

1. A process that ensures every element of the population has some known chance of being selected is known as

   A. Non- random sampling
   B. Simple random sampling
   C. Stratified sampling
   D. Random sampling
   E. Systematic sampling

2. Which of the following is **NOT** an example of non- probability sampling?

   A. Convenience sampling
   B. Quota sampling
   C. Cluster sampling
   D. Haphazard sampling
   E. Judgments sampling

3. Which of the following is **NOT** a bar chart?

   A. Component
   B. Simple
   C. Percentage component
   D. Cumulative
   E. Multiple

4. A distribution in which the mode is greater than the median and the median is greater than the mean is referred to as a

A Positively-skewed distribution
B Normal distribution
C Negatively-skewed distribution
D Continuous distribution
E Poisson distribution

5. Which of the following is normally used by an Accountant for making decision when series of values are summarised into a figure?

A. Average
B. Table
C. Chart
D. Regression
E. Correlation

6. Which of the following is **NOT** a measure of partitioning?

A. Quintile
B. Interquartile
C. Percentiles
D. Deciles
E. Quartiles

7. Which of the following is **NOT** a measure of dispersion?

A. Standard deviation
B. Interquartile range
C. Arithmetic mean
D. Mean deviation
E. Quartile Deviation

8. The following data shows the relationship between income (x) of an Accountant and the monthly upkeep given to the wife (y) for a period of 10 months: $\sum xy = 620,\ \sum x^2 = 440,\ \sum y^2 = 890,\ \sum x = 64,\ \sum y = 90$.

Calculate the Pearson's correlation coefficient .

A. 0.9412
B. 0.8412
C. 0.4899
D. 0.4889

E. 0.0016

9. A component of time series analysis that deals with a variation caused by unpredictable events such as floods, disasters, wars, etc is known as

A. Secular variation
B. Cyclical variation
C. Seasonal variation
D. Irregular variation
E Natural variation

10. The weighted index number that uses the arithmetic mean of the quantities or prices of the current and base time points as weighing factors is referred to

A. Fisher's ideal index
B. Laspeyre's index
C. Paasche's index
D. Marshal edge-worth's index
E. Simple aggregate price index

11. A survey of a housing estate showed that 28% of the tenants had GoTV and 78% had Startimes TV subscriptions. The probability that a household picked at random had either a GoTV or a Startimes TV subscription is

A 0.1584
B 0.1707
C 0.2184
D 0.7816
E 0.8416

12. The standard deviation of scores obtained by candidates who sat for Quantitative Analysis March diet of ATSWA examination was 15. If a random sample of 9 candidates gave a mean score of 47, then the calculated test statistic for testing the hypothesis: $H_0 : \mu = 50 \quad H_1 : \mu > 50$ is given as

A. t = -0.6
B. t = 0.6
C. t = 0.65
D. z = -0.6
E z = 0.6

13. If the revenue and cost functions (both in ₦) of a small firm producing a local hand sanitizer are respectively $C(x) = 5x + 2000$ and $R(x) = 9x$, where x is the quantity of the sanitizer produced and sold, then the maximum value of x for the firm to make a profit of at most ₦10,000 is

A. $x \leq 1{,}000$
B. $x \leq 2{,}000$
C. $x \leq 2{,}500$
D. $x \leq 3{,}000$
E. $x \leq 3{,}500$

14. The weekly costs (₦C) for AKJIL Plc were plotted against the company's production level (P) for the last 100 weeks. If the regression line for the company is estimated to be $C = 1{,}200 + 500$, which of the following statements is true about the weekly costs?

A. Fixed costs are ₦1,200. Variable costs per unit are ₦5
B. Fixed costs are ₦1,200. Variable costs per unit are ₦50,000
C. Fixed costs are ₦1,200. Variable costs per unit are ₦500
D. Fixed costs are ₦12. Variable costs per unit are ₦5
E. Fixed costs are ₦ 12. Variable costs per unit are ₦ 500

15. When there is a relationship between the change in the quantity demanded and the price of a good or service, the elasticity is known as

A. Income elasticity of demand
B. Gross elasticity of demand
C. Consumer elasticity of demand
D. Equilibrium elasticity of demand
E. Price elasticity of demand

16. A firm has estimated that the sales function $P(x) = 25x - 3$ and the cost function $C(x) = 1500 + 9x^2 - 13x$, where x is the number of items produced and sold. Determine the break-even quantity for the firm.

A. 16.0
B. 15.4
C. 15.0
D. 14.8
E. 10.7

17. Find the amount which would be obtained from an interest of ₦2,000 at 6% compounded quarterly for 5 years.

A. ₦2,593.71
B. ₦2,673.71
C. ₦2,683.71
D. ₦2,693.71
E. ₦2,793.71

18. The cash flows of a mini-project with the discount factors are presented in the table below:

Year	Net cash flow(₦)	Discount factor
0	(750,000)	1
1	200,000	0.833
2	450,000	0.694
3	550,000	0.579

The Net Present Value (NPV) of the project is

A. −₦43,750

B. −₦47,350
C. ₦43,750
D. ₦47,350
E. ₦50,000

19. Operations Research tools are from

A. Computer Science
B. Economics
C. Engineering
D. Mathematics
E. Statistics

20. A tight constraint will have a shadow price of

A Zero
B One
C Greater than zero
D The value of the right hand side of the constraint
E Less than zero

21. The objective function of Dual Linear Programming problem of the following Primal problem:

Min. P = $12x_1 + 26x_2$
Subject to: $3x_1 + 4x_2 \geq 5,$ and $2x_1 + 6x_2 \geq 6,$ $x_1, x_2 \geq 0$ is

A. Min.Q $= 5y_1 + 6y_2$
B. Max.Q $= 5y_1 + 6y_2$
C. Min.P $= 3x_1 + 4x_2$
D. Min.Q $= 3y_1 + 4y_2$
E. Min.P $= 5y_1 + 6y_2$

22. A firm's annual demand is 100,000 units. Each unit costs ₦400. If the cost of placing an order is ₦7,000 and the annual holding cost is 20% of the purchase price of a unit, calculate the economic order quantity

A. 4,181.3 units
B. 4,182.0 units
C. 4,183.3 units

D. 4,184.0 units
E. 4,284.3 units

23. Given that EOQ is 250,000 units in a manufacturing industry. If the cost of placing an order is ₦8,000 with the holding cost 10% of the purchase price per unit, determine the annual demand if the cost of each unit is ₦650.

A. 252,903,250 units
B. 252,904,250 units
C. 252,905,250 units
D. 253,906,250 units
E. 253,907,250 units

24. The formula for average stock level is given as

A. Minimum stock level + $\frac{1}{2}$ of Re-order level
B. Maximum stock level + $\frac{1}{2}$ of Re-order level
C. Minimum stock level + $\frac{1}{3}$ of Re-order level
D. Maximum stock level + $\frac{1}{3}$ of Re-order level
E. Minimum stock level + $\frac{1}{4}$ of Re-order level

Use the following information to answer questions 25 and 26:

The transportation problem of shipping commodities A, B and C to warehouses X, Y and Z is modeled with the unit costs tabulated below:

If the initial solution to the problem obtained using North-West Corner Rule (NWCR) method, is shown in the following allocation table:

Warehouses / Commodities	X	Y	Z	SUPPLY
A	2 **10**	3	4	15
B	1	2	2	20

		10	**Q**	
C	4	5	4 **R**	15
DEMAND	10	15	25	

25. The value of q is

A 25
B 20
C 15
D 10
E 5

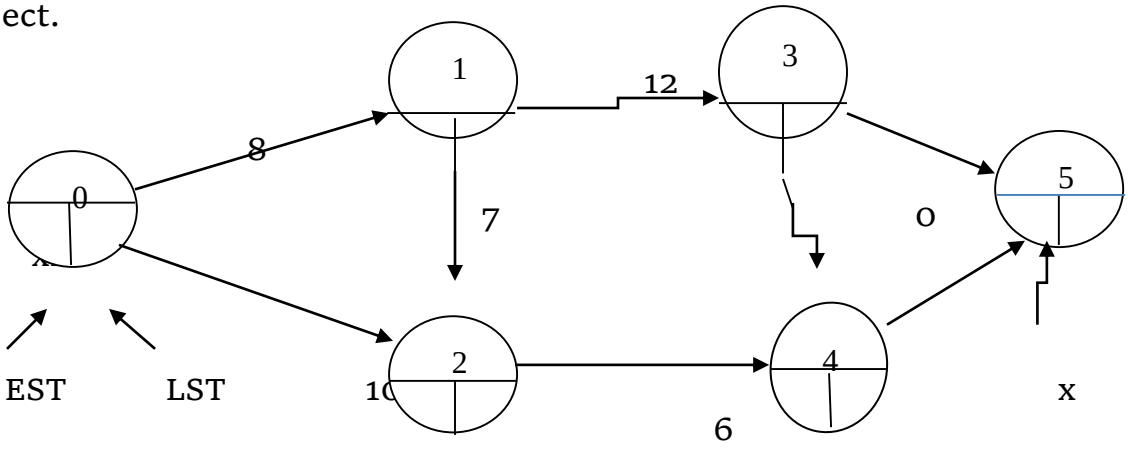
26. The value of r is

A 5
B 10
C 15
D 20
E 25

27. The amount of time,for which a group of activities could be delayed without affecting the overall project duration, is known as

A. Earliest time
B. Free float
C. Independent float
D. Latest time
E Total float

28. The Activity – On – Node network diagram below indicates the Earliest Starting Time (EST) and Latest Starting Time (LST) of a small project.



What is the value of the Earliest Starting Time, x?

A. 20
B. 23
C. 25
D. 33
E 43

29. If the average life span of an electronic component in an Automated Teller Machine (ATM) of a bank is 2.5, then the average number of weekly replacements of 600 components is

A. 150
B. 240
C. 850
D. 1040
E 1200

30. A firm manufactures 4 types of one of its products. The probability distributions for demanding these types of products are as tabulated below:

Type	Probability
A	0.15
B	0.42
C	0.28
D	0.15

Use the random numbers 63 and 38 to simulate the next two demands

A. Type C, Type D respectively
B. Type B, Type D respectively
C. Type D, Type C respectively
D. Type B, Type C respectively
E. Type C, Type B respectively

SECTION A: PART II SHORT-ANSWER QUESTION (20 MARKS)

ATTEMPT ALL QUESTIONS

Write the correct answer that best completes each of the following questions/statements

1. If a regression model $y = 4.5 - 6.2x,$ then the value of regression coefficient is .....

2. If the coefficient of variation of a data set is 100%, then its mean must be equal to its.....

3. The marks obtained by the students in an Accounting examination are 12, 11, 10, 7, 8, U and 16. If the mean is estimated to be 10, then the variance is .....

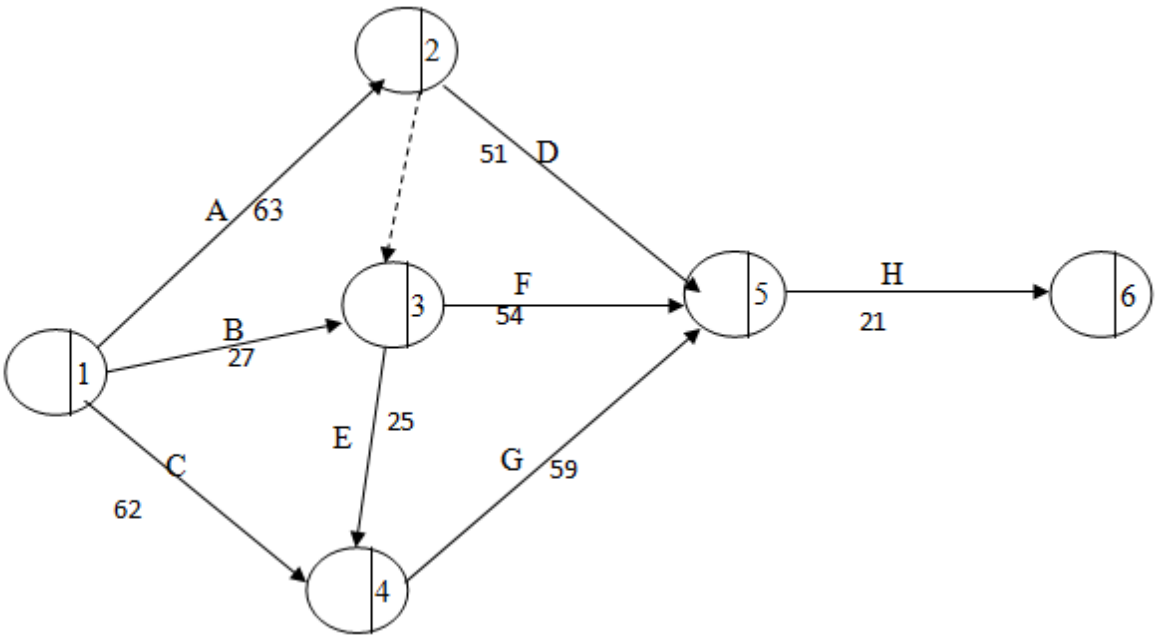
4. Given that the Laspeyre price index is 112.8% and Paasche price index is 113.69%, the Fisher's price index is .....

5. The present value of ₦1.8 million at 20% simple interest rate over $2\frac{1}{2}$ years is.....

6. The group of items in a stock at the time, during which inventory is taken is known as .....

7. For every Linear Programming (LP) problem, there is a corresponding dual form. The original LP problem is known as .....

Use the following network diagram to answer questions 8 and 9:



8. Find the sum of Earliest Start Times (EST) for activities F and H in weeks.

9. Calculate the Total float for Activity F in months if 30 days make a month.

10. The marks obtained by the students in an Accounting examination are 12, 11, 10, 7, 8, U and 16. If the mean is estimated to be 10, the median is .....

11. The selling price of an item produced by a company is ₦100. If the total overhead cost is ₦8,750 and the cost per unit is ₦75, then the number of units that must be produced and sold for the company to break-even is .....

12. A graphical solution to any form of equation is regarded as an .....

13. Currently, the number of methods for solving a Linear Programming problem is .....

14. The value of p in the table below is .....

Time (t)	**Value of series** (Y)	**Trend by LSM** $Y = 43 + 7t$	**Seasonal variation assuming additive model**
1	47		p
2	58		

3	62		

15. Rejection of a null hypothesis when it is true and expected to be accepted leads to .....

16. The two main costs that increase with time in the replacement of items that wear-out gradually are ..... and .....

17. Sample is a fractional part of a population for which data can be sought and it must be a .....sample.

18. Operations Research attempts to find ..... solution to a problem.

19. An investment has a net present value of ₦15,000 when the discount rate is 10%. If at the discount rate of 12 %, its net present value is ₦6,000, then the Internal Rate of Return (IRR) of this investment is ..... %

20. A transportation problem is unbalanced if ..... is not equal to .....

SECTION B: ATTEMPT ANY FOUR QUESTIONS (50 MARKS)

QUESTION 1

a. The following table shows the weekly wages of 50 members of staff in a factory:

Wage(Leo'000)	Number of Staff
5 – 10	14
11 – 20	7
21 – 30	15
31 – 40	8
41 -- 50	6

You are required to calculate the

i. Twentieth percentile (3½ Marks)

ii. Eightieth percentile (2½ Marks)

b. From the given observations below, establish the relationship among arithmetic mean, geometric mean and harmonic mean:

20, 18, 9, 7, 12, 16, 17, 24, 14 and 13. (6½ Marks)

(Total $12^{1}/_{2}$ Marks)

QUESTION 2

a.

Type of cost	Value (per year)
Cost of capital	8.50%
Cost due to breakages	6.50%
Rent paid toward physical space for storage	3.50%
Premium paid to insure inventory	0.25%
Tax	1.00%

The data in the table above shows

the data collected for one item.

Compute the following:

i. Inventory carrying rate (3 Marks)

ii. The annual carrying costs of an item that costs ₦20 (1½ Marks)

ii. The total annual carrying cost for 15 items that costs ₦20 per unit (1½ Marks)

iv. The total carrying cost for 15 items that cost ₦20 per unit and are held in inventory for a period of 2 years. (1½ Marks)

b. An item has an annual demand of 5,000 units, the inventory costs are based on an annual interest rate of 20%, the purchase cost of the item is ₦10 and the ordering cost is ₦25.20 per order.

Compute

i. The lot size for an item (2½ Marks)

iii. The cycle time, if there are 250 working days in a year (2½ Marks)

(Total 12$^1/_2$ Marks)

QUESTION 3

a. Determine the Quartile deviation of the hourly wage distribution of 500 workers of a manufacturing company given in the following table:

Class interval (₦'00)	Cumulative Frequency
10 - 19.9	11
20 - 29.9	35
30 - 39.9	89
40 - 49.9	239
50 - 59.9	347
60 - 69.9	425
70 - 79.9	476
80 - 89.9	496
90 - 99.9	500

(6$^1/_2$ Marks)

b. The distribution of book sales (in hundreds) in a bookshop is tabulated as follows:

Class Interval	25 - 29	30 - 34	35 - 39	40 – 44	45 - 49	50 - 54	55 – 59	60 - 64
Frequency	3	5	11	9	7	7	6	7

Calculate

i. the mean using a coding factor of 5 with an Assumed mean of 4700 ($3^1/_2$ Marks)

ii. the mean deviation ($2^1/_2$ Marks)

(Total $12^1/_2$ Marks)

QUESTION 4

a. A clothing manufacturing company produces jerseys for high school bookstores in runs of up to 200. If the manufacturer sells the jerseys at $Le 85$ each and the cost function for a run of x jerseys is estimated to be

$$C(x) = 1{,}550 + 10x + 0.3x^2 \qquad (0 \le x \le 200)$$

You are required to find:

i. How many clothing should the manufacturer produce to make profit ($3^1/_2$ Marks)

ii. The price at which the profit is maximised ($1^1/_2$ Marks)

iii. The maximum profit. (1 Mark)

iv. The price at maximum profit. (1 Mark)

b. The demand function for goods produced by a particular company is given by: $p(q) = \sqrt{2{,}500 - q}$,where q is the quantity of goods produced and sold and p (in Le) is the price of goods.

You are required to:

i. Investigate the effect of increase in price when 2,025 of the goods are demanded ($2^1/_2$ Marks)

ii. Comment on your result obtained in (i) above (1 Mark)

iii. Find the price that will maximise the revenue (2 Marks)

(Total 12$^1/_2$ Marks)

QUESTION 5

a. An organisation has three machine shops: A, B and C and it produces three products: X, Y and Z using these three machine shops. Each product involves the operation of the machine shops. The times available at the machine shops: A, B, and C are 100, 72 and 80 hours respectively. The profits of the products: X, Y and Z are ₦22, ₦6 and ₦2 per unit respectively. The following table shows the time required for each operation for unit quantity of each product:

Products	**Machine shops**		
	A	**B**	**C**
X	10	7	2
Y	2	3	4
Z	1	2	1

i. Formulate the appropriate Linear Programming Problem for minimising the profits and define all symbols used. (2$^1/_2$ Marks)

ii. Set up the initial tableau based on the simplex method (7$^1/_2$ Marks)

b. Obtain the dual problem of the following:

Maximise Z = 40a + 240b + 200c

Subject to $2a + 3b + 4c \leq 45$

$a + 8b + 5c \leq 30$

$a, b, c \geq 0$ (2$^1/_2$ Marks)

(Total 12$^1/_2$ Marks)

QUESTION 6

a. A sachet water dealer has 3 depots: X, Y and Z in 3 different locations. The dealer has 5,000 bags of sachet water in depot X, 4,500 bags in depot Y and 5,500 bags in depot Z. He has an order for his products from three different major distributors: D_1, D_2 and D_3. Distributor D_1 requires 5,600 bags, D_2 requires 6,700 bags and D_3 requires 3,700 bags. The table below shows the transportation cost per bag in Ghana cedis (GH₵) from each depot to each distributor:

	Distributors		
Depots	D_1	D_2	D_3
X	20	40	25
Y	30	50	40
Z	35	60	15

i Use the Vogel's Approximation Method to determine the initial basic number of bags that can be supplied from each depot to various distributors and hence, test for feasibility. (6 Marks)

ii Calculate the initial basic total transportation cost. ($1^1/_2$ Marks)

b. A businessman has 4 mini-projects to be assigned to 4 different contractors.

The table below gives the total cost (₦'million) of completing each project by the contractors based on their quotations:

	Projects			
Contractors	**X**	**Y**	**Z**	**W**
1	11	14	16	19
2	15	15	19	11
3	12	9	8	17
4	19	17	12	10

Use the reducing-cost matrix method to obtain the optimal assignment of the contractors to the projects and hence, determine the total minimum cost of the 4 projects. (5 Marks)

(Total $12^1/_2$ Marks)

FORMULAE

Sample variance, s^2 = $\dfrac{\sum(x-\bar{x})^2}{n-1}$

Economic Order Quantity

Q = $\sqrt{\dfrac{2cd}{n}}$

Z_{cal} = $\dfrac{\bar{x}-\mu}{\dfrac{\sigma}{\sqrt{n}}}$

Slope of a regression equation

b = $\dfrac{n\sum xy-\sum x\sum y}{n\sum x^2-\left(\sum x\right)^2}$

Elasticity of demand, $e=\left(-\dfrac{p}{q}\right)\left(\dfrac{dq}{dp}\right)$

The 95% confidence interval for μ

$= \bar{x} \pm t_{\frac{\alpha}{2},n-1}\dfrac{s}{\sqrt{n}}$

The trend equation, y = a + bt, where t = x_i - x_m

b = $\dfrac{\sum ty}{\sum t^2}$. **a =** $\bar{y}-bx_m$, **x_m = median of x values**

SARPI = $\dfrac{\sum\left(\dfrac{P_n}{P_o}\times 100\right)}{N}$

SAPI = $\dfrac{\sum P_{ni}}{\sum P_{oi}}\times 100$

$t = \dfrac{p}{\sqrt{\dfrac{pq}{n}}}$

EOQ with stock-out

$$Q = \sqrt{\frac{2cd}{h}} \times \sqrt{\frac{h + c_s}{c_s}}$$

$$LPI = \frac{\sum p_1 q_o}{\sum p_o q_o} \times 100$$

$$Z = \frac{p - \hat{p}}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

$$Q_i = L_{Q_i} + \left(\frac{\frac{iN}{4} - \sum f_{Q_i}}{f_{Q_i}} \right) c$$

$$D_i = L_{D_i} + \left(\frac{\frac{iN}{10} - \sum f_{D_i}}{f_{D_i}} \right) c$$

$$P_i = L_{P_i} + \left(\frac{\frac{iN}{100} - \sum f_{P_i}}{f_{P_i}} \right) c$$

Spearman's rank correlation coefficient

$$r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

EOQ with gradual replenishment

$$Q = \sqrt{\frac{2cd}{h\left(1 - \frac{d}{r}\right)}}$$

$$\text{Length of Inventory cycle} = \frac{Q}{d}$$

$$\text{Number of } productionruns = \frac{d}{Q}$$

$$\text{Production cost} = Ordering\,\text{cost} + Holdering\,\text{cost}$$

$$Mode = L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2} \right) c$$

SOLUTION TO QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. D
2. C
3. D
4. C
5. A
6. B
7. C
8. B
9. D
10. D
11. E
12. D
13. D
14. C
15. E
16. C
17. D
18. D
19. E
20. C
21. B

22. D

23. D

24. A

25. D

26. C

27. E

28. C

29. B

30. E

WORKINGS (MCQ)

8. $$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2 - (\sum x)^2][n(\sum y^2 - (\sum y)^2]}} = \frac{10(620) - 64(90)}{\sqrt{[10(440) - (64)^2][10(900) - (90)^2]}}$$

$$= \frac{6200 - 5760}{\sqrt{[4400 - 4096][9000) - 8100]}} = \frac{440}{\sqrt{[304][900]}} = \frac{440}{\sqrt{273600}}$$

$$= \frac{440}{523.07} = 0.8412$$

Answer is B. 0.8412

7. Pr (Household has either a GoTV or a Startimes TV)

= Pr (GoTV) + Pr (Startimes TV)- Pr (both GoTV and Startimes TV)

$$= 0.28 + 0.78 - (0.28 \times 0.78)$$
$$= 1.026 - 0.2184$$

$$= 0.8416$$

The correct option is **E**

12. *Since δ is known i.e* $\delta = 15$

The test statistic is

$$Z = \frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}$$

$$Z = \frac{47 - 50}{\frac{15}{\sqrt{9}}} = \frac{-3}{5}$$

$$Z = -0.6$$

13. $\text{Pr}ofit\ \ function, P(x) = R(x) - C(x)$

$$P(x) = 9x - (5x + 2{,}000)$$

$$P(x) = 4x - 2{,}000$$

$$but\ P(x) \le 10{,}000$$

$$\therefore 4x - 2{,}000 \le 10{,}000$$

$$4x \le 10{,}000 + 2{,}000$$

$$4x \le 12{,}000$$

$$x \le \frac{12{,}000}{4}$$

$$x \le 3{,}000$$

(D)

14. If $C = 1{,}200 + 500$, then the fixed costs are ₦ 1,200 and variable costs are ₦500 per unit. **C**

16. TR = P(x) X x = (25x- 3)x = $25x^2$-3x

TC = 1500 + $9x^2$ - 13x

At breakeven point TR = TC

$25x^2$-3x = 1500 + $9x^2$ - 13x

$25x^2$- $9x^2$-3x + 13x + 1500 = 0

$16x^2$ + 10x + 1500 = 0

x = 15 or -16

Since x cannot be negative, x = 15units

Break-even quantity for the firm is 15units

x=15 (B)

17. $P = 2000, r = 6\% = \frac{0.06}{4} = 0.015$

$n = 5 \times 4 = 20$

$A = P(1+r)^n = 2000(1+0.015)^{20} = N2693.71 \qquad (D)$

18.

$NPV = (-750{,}000 \times 1) + (200{,}000 \times 0.833) + (450{,}000 \times 0.694) + (550{,}000 \times 0.579)$

$NPV = -750{,}000 + 166{,}600 + 312{,}300 + 318{,}450$

$NPV = ₦47{,}350 \qquad (D)$

22. $C_0 = \#7000, C_h = \frac{20}{100} \times 400 = \#80, D = 100{,}000$

$EOQ = \sqrt{\frac{2C_0D}{C_h}} = \sqrt{\frac{2 \times 7000 \times 100{,}000}{80}} = 4{,}183.3$ units = 4,184 unit (D)

23. $EOQ = 250{,}000\ C_0 = 8000, C_h = \frac{10}{100} \times 650 = \#65$

$250{,}000 = \sqrt{\frac{2 \times 8000 \times D}{65}}$

$D = \frac{250{,}000^2 \times 65}{16{,}000} = 253{,}906{,}250$ units (D)

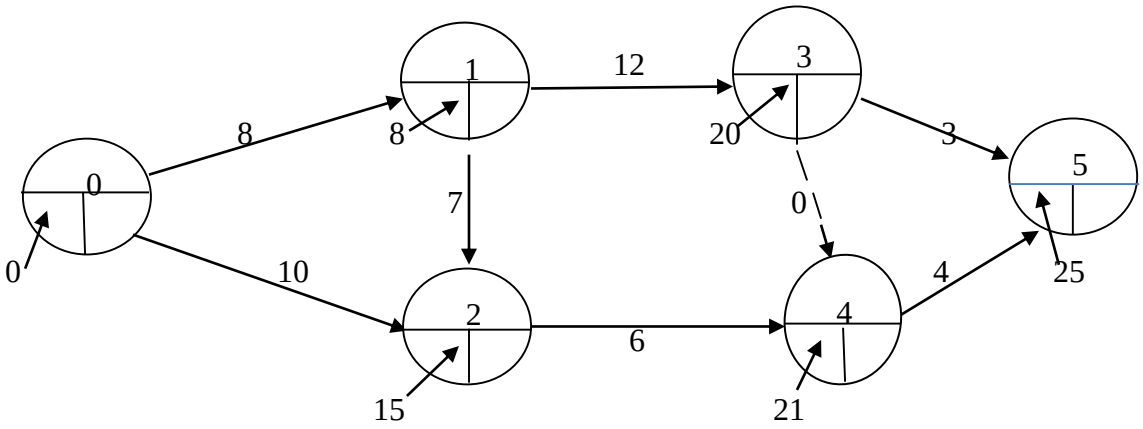
To answer questions25 and 26, the allocation table is completed, using NWCR method, as follows:

	X			
	Y	*Z*	Supply	
A	2 **10**	3 **5**	4	~~15~~ ~~10~~ ~~5~~ ~~0~~
B	1	2 **10** 2	2 **10** 2	~~20~~ ~~10~~ ~~0~~
C	4	5	4 15	~~15~~
Demand	~~10~~ ~~0~~	~~15~~ ~~5~~ ~~0~~	~~25~~ ~~10~~ ~~5~~ ~~0~~	

25. From the allocation table, the value of $q = 10$. **D**

26. From the allocation table, the value of $r = 15$. **C**

28.



To get the EST, by using the forward pass approach

At node 0, EST = 0

At node 1, EST = 8

At node 2, EST = maximum (10, 8+7=15) = 15

At node 3, EST = 8+12 = 20

At node 4, EST = maximum (20+0=20, 15+6=21) = 21

At node 5, EST = maximum (20+3=23, 21+4=25) = 25

Therefore, the Earliest Start Time, x = 25 (C)

29.

$$Average\,number\,of\;\;weekly\,replacements = \frac{Number\,of\;\;component}{Average\,life\,span\,of\;\;component}$$

$$Average\,number\,of\;\;weekly\,replacements = \frac{600}{2.5}$$

$$Average\,number\,of\;\;weekly\,replacements = 240 \qquad \text{(B)}$$

30.

Type	Probability
A	0.15
B	0.42
C	0.28
D	0.15

Random numbers (R-N) are 63, 35,

For R-N 63 is Type C

For R-N 35 is Type B (E)

SHORT ANSWER QUESTIONS

1. −6.2
2. Standard deviation
3. 10
4. 113.24%
5. ₦ 1.2million
6. Physical stock
7. Primal Problem
8. 30weeks
9. 1 month
10. amedian = 10
11. 350
12. Estimate/Approximate solution/Approximation
13. Three methods
14. p = -3
15. Type I error
16. Maintenance , repair
17. Representative sample
18. Optimum
19. 13.33 %
20. Total demand , total supply

WORKINGS (SAQ)

3. $Mean = \frac{\sum X}{n} = 10$

where $n = 7$

$\sum X = 12 + 11 + 10 + 7 + 8 + U + 16$

$\sum X = 64 + U$

$\therefore 10 = \frac{64 + U}{7}$

$70 = 64 + U$

$U = 70 - 64$

$U = 6$

Using $Variance = (S.D)^2$

where $SD = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

$Variance = \frac{\sum (x - \bar{x})^2}{n}$

where $n = 7 \; and \; \bar{x} = 10$

$\sum (x - \bar{x})^2 = (12 - 10)^2 + (11 - 10)^2 + (10 - 10)^2 + (7 - 10)^2 + (8 - 10)^2 + (6 - 10)^2 + (16 - 10)^2$

$\sum (x - \bar{x})^2 = 4 + 1 + 0 + 9 + 4 + 16 + 36$

$\sum (x - \bar{x})^2 = 70$

$Variance = \frac{70}{7}$

$Variance = 10$

4.

$$Fisher'sPriceIndex = \sqrt{Laspeyrepriceindex \times PaaschePriceindex}$$

$$Fisher'sPriceIndex = \sqrt{112.8 \times 113.69} = 113.24\%$$

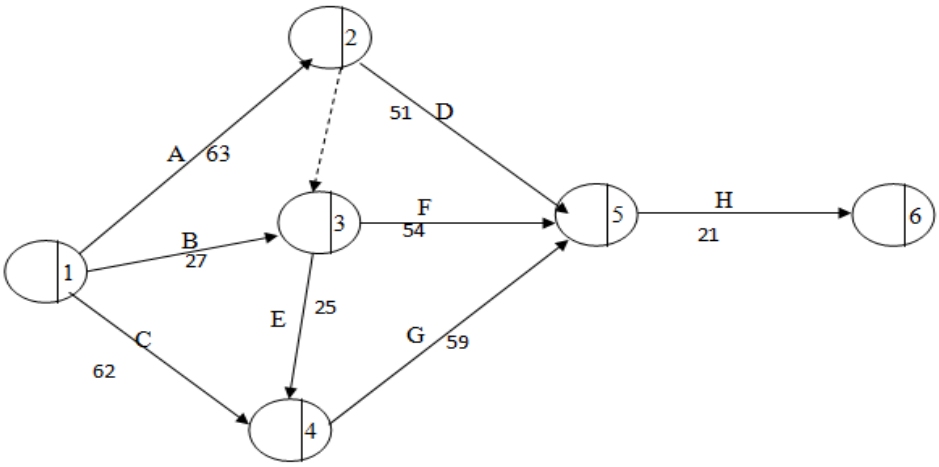
5. $A = P(1+rn)$

where $A = 1.8m$, $r = 20\% = 0.2$, $n = 2\frac{1}{2}\,years = 2.5$

$$P = \frac{A}{1+rn} = \frac{1.8m}{1+(0.2)(2.5)} = \frac{1.8m}{1.5} = 1.2m$$

This implies that the present value is ₦ 1.2million

8.



The sum for Earliest SartingTimesfor Activity F = 0 + 63 + 0 = 63 days

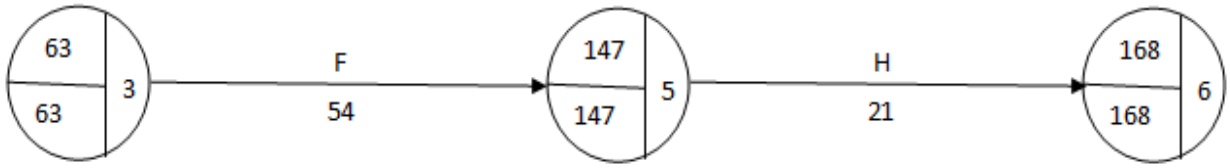
The sum for Earliest SartingTimesfor Activity H = 0 + 63 + 0 + 25 + 59 = 147days

Sum of Earliest Start Time (EST) for activities F and H = 63 + 147= 210 days = 30 weeks

Answer is : 30 weeks

9 Total Float = Latest Finish Time (LFT) - Earliest Start Time (EST) - Duration (D)

Total Float = LFT - EST - D



Total Float for activity F = 147 - 63 - 54 = 30 days = 1 month (since 30days make a month)

Answer is : 1 month

10. $Mean = \frac{\sum X}{n} = 10$

where $n = 7$

$$\sum X = 12 + 11 + 10 + 7 + 8 + U + 16$$
$$\sum X = 64 + U$$

$$\therefore 10 = \frac{64 + U}{7}$$

$$70 = 64 + U$$
$$U = 70 - 64$$
$$U = 6$$

In descending order, the data is arranged as follows: 16, 12, 11, 10, 8, 7, 6

Therefore, the median = 10

11. *Let x be the number of units to be produced and sold*

Total Re*venue Function,* $R(x) = 100x$

Total Cost Function, $C(x) = 75x + 8{,}750$

For the company, to breakeven

$$R(x) = C(x)$$

$$100x = 75x + 8{,}750$$

$$100x - 75x = 8{,}750$$

$$25x = 8{,}750$$

$$x = \frac{8{,}750}{25} = 350$$

14. $p = 47 - (43 + 7(1))$

$p = 47 - 50$

$p = -3$

19. From the relation $IRR = a\% + \left[\frac{NPV_A}{NPV_A - NPV_B} \times (b - a)\right]\%$,

where $a = 10,\ NPV_A = 15{,}000\ NPV_B = 6{,}000$ and $b = 12.$

$$\therefore IRR = 10\% + \left[\frac{15{,}000}{15{,}000 - 6{,}000} \times (12 - 10)\right]\%$$

$$= 10\% + (3.3333333333)\%$$

$$= 13.3333333333$$

$$\approx 13.33\%$$

QUESTION 1

a.i.

Wage	Frequency	Class boundary	Cumm.freq (cf)
5-10	14	4.5-10.5	14
11-20	7	10.5-20.5	21
21-30	15	20.5-30.5	36
31-40	8	30.5-40.5	44
41-50	6	40.5-50.5	50
	50		(1mk)

$$p_{20} = L_{p20} + \left(\frac{\frac{20}{100} N - Cfb_{p20}}{f_{p20}} \right) C_{p20}$$

$$where\ L_{p20} = 4.5,\ Cfb_{p20} = 0,\ \ f_{p20} = 14, C_{p20} = 10.5 - 4.5 = 6, \frac{20}{100} \times 50 = 10 \quad (1mk)$$

$$p_{20} = 4.5 + \left(\frac{10-0}{14} \right) \times 6 \qquad (1mk)$$

$$= 4.5 + 4.29$$

$$= 8.79 \qquad (1/2mk)$$

aii.

$$p_{80} = L_{p80} + \left(\frac{\frac{80}{100} N - Cfb_{p80}}{f_{p80}} \right) C_{p80}$$

$$where\ L_{p80} = 30.5,\ Cfb_{p80} = 36,\ \ f_{p80} = 8, C_{p80} = 40.5 - 30.5 = 10, \frac{80}{100} \times 50 = 40 \quad (1mk)$$

$$p_{80} = 30.5 + \left(\frac{40-36}{8}\right)\times 10 \qquad (1mk)$$

$$= 30.5 + 5$$

$$= 35.5 \qquad (1/2mk)$$

b. Arithmetic mean $= \dfrac{\sum x}{n}$

$$= \frac{20+18+9+7+12+16+17+24+14+13}{10} \qquad (1/2mk)$$

$$= \frac{150}{10} = 15 \qquad (1/2mk)$$

Harmonic mean

$$HM = \frac{n}{\sum \frac{1}{n}} = \frac{10}{0.05+0.056+0.111+0.143+0.083+0.063+0.059+0.042+0.071+0.077}$$

(1/2mk)

$$= \frac{10}{0.755} \qquad (1mk)$$

= 13.25 (1mk)

Geometric mean

$$GM = \sqrt[10]{(20)(18)(9)(7)(12)(16)(17)(24)(14)(13)} \qquad (1/2mk)$$

$$= \sqrt[10]{3233522074\times 10^{11}}$$

$$= \left(3233522874\times 10^{11}\right)^{\frac{1}{10}} \qquad (1mk)$$

$$= 14.16 \qquad (1mk)$$

$\therefore HM < GM < AM$ (1/2mk)

QUESTION 2

a.

i. The carrying rate is an aggregation of:

- Cost of capital (8.50%);
- Cost of storage (3.50%);
- Cost of inventory risk (6.50%);
- Cost of inventory servicing (0.25% + 1.00%)

(1½)

Therefore, the carrying rate is

= 8.50 + 3.50 + 6.50 + 0.25 + 1.00 (½) = 19.75% per year(1)

ii. The annual carrying costs of an item that cost₦20 per unit. Which is

$C_h = \frac{19.75}{100} \times 20$ (½) = ₦3.95(1)

The carrying cost is ₦3.95 per unit per year.

For C_h= ic

Where C_h= carrying cost or hold cost

i = The inventory carrying rate

c = The unit price cost of the item.

iii. The total annual carrying cost of 15 items that cost ₦20 per unit is

$$C_h = \frac{19.75}{100} \times 20 \times 15 \text{ (½)} = ₦59.25\text{(1)}$$

The total carrying cost is ₦59.25 per year

iv. The total carrying cost for 15 items that cost ₦20 per unit and are held in inventory for a period of 2 years would be:

$$C_h = \frac{19.75}{100} \times 20 \times 15 \times 2 \text{ (½)} = ₦118.50 \text{(1)}$$

The total carrying cost over a 2 year period is ₦118.50

b. For the time units for demand and carrying cost are the same. Other information provided are as follows:

- D is 5000 per year
- i is 0.20
- C is ₦10 per item
- C_o is ₦25.20 per order

(1)

i. For Q = $\sqrt{\frac{2DC_0}{ic}} = \sqrt{\frac{2\times 5000\times 25.20}{0.2\times 10}}$ (½) $= 355 units$ (1)

The EOQ for the item is 355 unit.

ii. $For\ cycle\ time = t\ and\ t = \frac{Q}{D} = \frac{355}{5000}$ (½) $= 0.071 year$ (½)

$Since\ there\ are\ 250\ days\ in\ a\ year, the\ cycle\ time$

$= 0.071 year\ x\ 250 \frac{days}{year}$ (½) $=$ $17.8 days$ (1)

(Total 12 $\frac{1}{2}$ marks)

QUESTION 3

a.

Class interval (₦)	F	CF	Class boundary
10 - 19.9	11	11	9.95 - 19.95
20 - 29.9	24	35	19.95 - 29.95
30 - 39.9	54	89	29.95 - 39.95
40 - 49.9	150	239	39.95 - 49.95
50 - 59.9	108	347	49.95 - 59.95
60 - 69.9	78	425	59.95 - 69.95
70 - 79.9	51	476	69.95 - 79.95
80 - 89.9	20	496	79.95 - 89.95
90 - 99.9	4	500	89.95 - 99.95
	500 (½ mark)		(½ mark)

Quartile deviation = $\frac{Q_3 - Q_1}{2}$ (½ mark)

$$Q_3 = L_{Q_3} + \left(\frac{\frac{3N}{4} - \sum f_{Q_3}}{f_{Q_3}} \right) C, ,$$

$Q_3 : \frac{3N}{4} = \frac{3X\,500}{4} = 375,\ L_{Q_3} = 59.95,\ \sum f_{Q_3} = 347,\ f_{Q_3} = 78, C = 10$ (1 mark)

$Q_3 = 59.95 + \left(\frac{375 - 347}{78} \right) 10$ (½ mark)

= 59.95 + 3.5897 = 63.5397 ≈ ₦63.54 (½ mark)

$$Q_1 = L_{Q_1} + \left(\frac{\frac{N}{4} - \sum f_{Q_1}}{f_{Q_1}} \right) C,$$

$Q_1 : \frac{N}{4} = \frac{500}{4} = 125,\ L_{Q_1} = 39.95,\ \sum f_{Q_1} = 89,\ f_{Q_1} = 150, C = 10$ (1mark)

$Q_1 = 39.95 + \left(\frac{125 - 89}{150} \right) 10$ ($\frac{1}{2}$ mark)

= 39.95 + 2.4 = 42.35 = ₦42.35 ($\frac{1}{2}$ mark)

Quartile deviation = $\frac{63.54 - 42.35}{2} = \frac{21.35}{2}$ ($\frac{1}{2}$ mark)

= $10.595 \approx$ ₦1060.00k ($\frac{1}{2}$ mark)

b

C I	F	X	U	FU	$X - \overline{X}$	$\lvert X - \overline{X} \rvert$	F$\lvert X - \overline{X} \rvert$
25-29	3	27	-4	-12	-18.36	18.36	55.08
30-34	5	32	-3	-15	-13.36	13.36	66.8
35-39	11	37	-2	-22	-8.36	8.36	91.96
40-44	9	42	-1	-9	-3.36	3.36	30.24
45-49	7	47	0	0	1.64	1.64	11.48
50-54	7	52	1	7	6.64	6.64	46.48
55-59	6	57	2	12	11.64	11.64	69.84
60-64	7	62	3	21	16.64	16.64	116.48 (1mk)
	55 (1/2mk)	(1/2mk)		-18 (1/2mk)	(1/2mk)		488.36 (1/2mk)

(i) Mean = A + $\left(\frac{\sum FU}{\sum F}\right)C$ (1/2 mark)

= 47 + $\left(\frac{-18}{55}\right)5$ (1/2 mark)

= 47 - 1.6364 = 45.3636 $\approx$4536

Therefore, the MEAN book sales = 4,536 (1/2 mark)

(ii) Mean Deviation = $\frac{\sum F\left|X-\overline{X}\right|}{\sum F}$ = $\frac{488.36}{55}$ = 8.879 (1/2 mark)

Therefore, Mean Deviation $\approx$ 888 book sales (1/2 mark)

Total marks = (12 $\frac{1}{2}$ marks)

QUESTION 4

a. i. Profit function = Revenue function – Cost function

i.e $P(x) = R(x) - C(x)$

$P(x) = 85x - (1{,}550 + 10x + 0.3x^2)$ $\frac{1}{2}$

At break-even point, $P(x) = 0$.

$85x - (1{,}550 + 10x + 0.3x^2) = 0$ $\frac{1}{2}$

$75x - 1{,}550 - 0.3x^2 = 0$

where $x = \dfrac{75 \pm \sqrt{(75)^2 - 4 \times 0.3 \times 1550}}{2 \times 0.3}$ $\frac{1}{2}$

$x = \dfrac{75 \pm \sqrt{3{,}765}}{0.6}$

$x = \dfrac{75 \pm 61.34}{0.6}$

$x = 227.2$ and $x = 22.8$ 1

But $0 \le x \le 200$,

$\therefore x = 22.8$ $\frac{1}{2}$

Hence, in order to make a profit, 23 or more clothing must be produced. $\frac{1}{2}$

ii. At the turning point (for maximum, minimum or inflection point),

$\dfrac{dP}{dx} = 0$ $\frac{1}{2}$

$\therefore 75 - 0.6x = 0$

$x = \dfrac{75}{0.6} = 125$ $\frac{1}{2}$

$\dfrac{d^2P}{dx^2} = -0.6 < 0$. This shows that the profit is maximum when the quantity $x = 125$

. $\frac{1}{2}$

iii. Maximum profit $P(125) = 75 \times 125 - 1550 - 0.3 \times 125$ $\frac{1}{2}$

$P(125) = 75 \times 125 - 1{,}550 - 0.3 \times 125.$

$P(125) = 9{,}375 - 1{,}550 - 37.5 = 7.785$ $\frac{1}{2}$

iv. The price at maximum profit can be obtained from

$$p(125) = \frac{R(125)}{125} \quad \tfrac{1}{2}$$

$$p(125) = \frac{85 \times 125}{125}$$

$$p(125) = Le\ 85 \quad \tfrac{1}{2}$$

b. i. Elasticity of demand is

$$\eta = \left(\frac{-p}{q}\right) \times \left(\frac{dq}{dp}\right)$$

where $p = \sqrt{2{,}500 - q}$,

$$q = 2{,}500 - p^2$$

$$\frac{dq}{dp} = -2p \quad \tfrac{1}{2}$$

$$\therefore \eta = \left(\frac{-p}{q}\right) \times (-2 \times p) \quad \tfrac{1}{2}$$

When $q = 2{,}025$,

$$p = \sqrt{2{,}500 - q}$$

$$p = \sqrt{2{,}500 - 2{,}025} \quad \tfrac{1}{2}$$

$$p = 21.79$$

$$\eta = \left(\frac{-21.79}{2{,}025}\right) \times (-2 \times 21.79) \quad \tfrac{1}{2}$$

$$\eta = \frac{950.24}{2{,}025} = 0.47 \quad \tfrac{1}{2}$$

(ii) Since the elasticity of demand, $\eta = 0.47 < 1$, this shows that it is an inelastic demand and at a price of Le21.79, a 1% increase in price would decrease demand by only 0.47%. 1

(iii) To maximize revenue, we solve for p when $\eta = 1$. $\tfrac{1}{2}$

$$\therefore \eta = \left(\frac{-p}{q}\right) \times \left(\frac{dq}{dp}\right) = 1 \quad \tfrac{1}{2}$$

$$q = 2{,}500 - p^2$$

$$\frac{dq}{dp} = -2p$$

$$\left(\frac{-p}{2{,}500 - p^2}\right) \times (-2p) = 1 \qquad \frac{1}{2}$$

$$2{,}500 - p^2 = 2p^2$$

$$p^2 = \frac{2{,}500}{3}$$

$$p = \sqrt{833.33}$$

$$p = Le28.87 \qquad \frac{1}{2}$$

QUESTION 5

a.

i Let x_1 be number of units of product X that will be produced

Let x_2 be number of units of product Y that will be produced

Let x_3 be number of units of product Z that will be produced

We have to develop a linear programming formulation. The linear programming formulation of the product mix problem is:

Maximize: $22x_1 + 6x_2 + 2x_3$ (½)

Subject to: $10x_1 + 2x_2 + x_3 \leq 100$

$7x_1 + 3x_2 + 2x_3 \leq 72$ (2)

$2x_1 + 4x_2 + x_3 \leq 80$

$x_1, x_2, x_3 \geq 0$

ii. We introduce slack variables $S_4, S_5 and S_6$ to make the inequality become equation.

Thus, the problem can be stated as

Maximize: $22x_1 + 6x_2 + 2x_3$

Subject to: $10x_1 + 2x_2 + x_3 + S_4 = 100$ (½)

$7x_1 + 3x_2 + 2x_3 + S_5 = 72$ (½)

$2x_1 + 4x_2 + x_3 + S_6 = 80$ (½)

$x_1, x_2, x_3, S_4, S_5, S_6 \geq 0$ (½)

From the above equation, the Simplex table can be obtained

	Basic	C_1	22	6	2	0	0	0
CB	Variables	XB	X_1	X_2	X_3	S_4	S_5	S_6
0	S_4	100	10	2	1	1	0	0
0	S_5	72	7	3	2	0	1	0
0	S_6	80	2	4	1	0	0	1
	$Z_j - C_j$		- 22	- 6	- 2	0	0	0

(5 ½ marks)

(-½ e.e)

b. Let the dual variables be x_1, x_2; then the dual problem is

Minimize $Z = 45x_1 + 30x_2$ (½)

Subject to: $2x_1 + x_2 \geq 40$ (½)

$3x_1 + 8x_2 \geq 240$ (½)

$4x_1 + 5x_2 \geq 200$ (½)

$x_1, x_2 \geq 0$ (½)

(12 ½ marks)

QUESTION 6

a. i. Steps in constructing the tableau by Vogel's Approximation Method:

- Calculate the penalty for each row & column which is the difference between the two least costs in the row & column
- Chosen the row/column with the highest penalty and allocate maximum possible to the least cost in the row/column chose
- Ignore the row/column in 2nd bullet and repeat until all allocations are satisfied

(Source/origin) Depots	(Destination) Distributors			(Availability) Supply	Row penalty
	M_1	M_2	M_3		
X	20 2,800	40 2,200	25 _	~~5,000~~ ~~2200~~ 0	5,20,20
Y	30 _	50 4,500	40 _	~~4,500~~ 0	10,20,20
Z	35 1,800	60 _	15 3,700	~~5,500~~ ~~1,800~~ 0	20,25,-
Demand	~~4,600~~ ~~2,800~~ 0	~~6,700~~ ~~4,500~~ 0	~~3,700~~ 0	15,000	
Column Penalty	10,10,10	10,10,10	10.-, -		

(5 *marks*)

Number of rows, n = 3, Number of columns, m = 3

Test for feasibility using n + m – 1 = filled cells

n + m – 1 = 2 + 3 -1 = 5

Since the number of filled cells = 5 = n + m – 1, therefore, the solution by Vogel's Approximation Method is feasible (1 *mark*)

ii. Initial basic total transportation cost = (2,800× 20) + (2,200× 40) + (4,500× 50) + (1,800×35) + (3,700×15) ($\frac{1}{2}$ *mark*)

= 56,000 + 88,000 + 225,000 + 63,000 + 55,500 ($\frac{1}{2}$ *mark*)

= GH₵487,500 ($\frac{1}{2}$ *mark*)

($7\frac{1}{2}$ *marks*)

b.

	Projects			
Contractors	X	Y	Z	W
1	11	14	16	19
2	15	15	19	11
3	12	9	8	17
4	19	17	12	10

Step (i): Subtract row minima i.e. subtract the smallest entry in each row from each entry in that row

	Projects			
Contractors	X	Y	Z	W
1	0	3	5	8
2	4	4	8	0
3	4	1	0	9
4	9	7	2	0

($\frac{1}{2}$ *mark*)

Step (ii): Subtract column minima i.e. subtract the smallest entry in each column from each entry in that column

	Projects			
Contractors	X	Y	Z	W
1	0	2	5	8
2	4	3	8	0
3	4	0	0	9
4	9	6	2	0

$(\frac{1}{2}\,mark)$

Step (iii): Cover all zeros with the minimum number of lines

	Projects			
Contractors	X	Y	Z	W
1	0	2	5	8
2	4	3	8	0
3	4	0	0	9
4	9	6	2	0

$(\frac{1}{2}\,mark)$

Since the number of lines is not equal to number of rows/columns

Step (iv): Create additional zero by subtracting the smallest element that isn't covered by a line from all the uncovered elements and add it to any element that is covered twice

	Projects			
Contractors	X	Y	Z	W
0	0		3	8
1	4		6	0
0	6		0	11
4	9		0	0

$(1\,mark)$

Since the number of lines = number of rows/columns

This implies that optimal assignment can be made:

Assign project X to contractor 1 $(\frac{1}{2}\,mark)$

Assign project Y to contractor 3 $(\frac{1}{2}\,mark)$

Assign project Z to contractor 4 $(\frac{1}{2}\,mark)$

Assign project W to contractor 2 $(\frac{1}{2}\,mark)$

Total cost of the 4 projects =

Project X to contractor 1will cost ₦11million

Project Y to contractor 3 will cost ₦9million

Project Z to contractor 4 will cost ₦12million

Project W to contractor 2 will cost ₦11million

Total = ₦43million $(\frac{1}{2}\,mark)$

$(5\ marks)$

$(Total\ 12\frac{1}{2}\,marks)$